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Consistency between Sea Surface Reconstructions from Nautical X-Band 
Radar Doppler and Amplitude Measurements 
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ABSTRACT

This study comprises the analysis and the interpretation of the coherent and the noncoherent parts of a coherent-on-receive microwave radar at grazing incidence conditions. The Doppler measurement is an extension of standard civil marine radar technology. While intensity images require interpretation based on understanding the underlying imaging mechanism, the Doppler signal measures the motion of an area of sea surface and is therefore closely related to the wave physics. Both the measured Doppler signal and the backscatter intensity signal are suitable for surface inversion and give almost identical surface elevations. A statistical comparison with a nearby buoy showed good correlation for the significant wave height and the peak period. By comparing the Doppler signal and the amplitude in the backscatter, the study amends the understanding of imaging mechanisms in marine radars at grazing incidence.

1. Introduction

Marine X-band radars are used commercially to monitor sea-state parameters, such as significant wave height ($H_s$), mean and peak periods and wavelengths, mean and peak directions, near-surface currents, etc., as well as wave spectra (Nieto-Borge et al. 1999; Hessner et al. 2001; Senet et al. 2001; Lund et al. 2014). Marine radars are attractive because of their availability on every moving vessel and on many on- and offshore platforms as well as their potential in providing the evolution of the sea surface in the spatiotemporal domain (Young et al. 1985; Hessner et al. 2001). However, intensity images are influenced by a number of known and unknown effects modulating the backscattered signal (Plant et al. 1978; Alpers and Hasselmann 1978; Plant et al. 2010; Lund et al. 2014). Hence, prior to deriving sea-state parameters from intensity images, the radar must be calibrated and image processing algorithms must be applied. A calibration campaign is normally assisted by in situ sensors, such as wave buoys (Nieto Borge 1998), providing time series of the wave elevation, wave slopes, or horizontal displacements, etc. that are considered reliable (Ochi 2008; Goda 2010; Cornejo-Bueno et al. 2016). The image processing is mainly composed of noise filtering and applying a modulation transfer function (MTF) that transfers the intensity images to a surface representation (Young et al. 1985; Nieto Borge and Guedes Soares 2000; Nieto Borge et al. 2004). While the applicability of this approach has been proven in a number of studies, its general validity remains uncertain.

In recent years there have been some efforts on improving the radar technology for marine and offshore...
applications. One attempt is to determine the phase of the returned backscattered signal to derive its Doppler frequency and thus the scattering elements’ velocity, which is related to the motion of the backscattering elements (Carrasco et al. 2017b), that is, Bragg scattering and microbreakers (Alpers et al. 1981; Plant and Keller 1990; Lee et al. 1995). In contrast to the intensity images that require additional interpretation, the Doppler measurement has a direct relation to the periodic components resulting from the orbital velocities of the waves. In addition to the orbital velocities, the Doppler velocity includes static contributions, such as currents, wind drift, and the phase speed of the resonant Bragg waves (Plant 1997). Assuming that the Doppler signal and the intensity signal are independent measurements of the same wave field scanned by the radar antenna, a comparison of the two magnitudes may help to understand the imaging mechanisms. This idea was applied in a number of studies, focusing on measuring and explaining the modulus of the MTF between the backscattered signal to derive its Doppler frequency resulting from the Doppler velocity derived from the coherent radar measurements used in this work. Section 4 describes the proposed method to estimate waves based on the Doppler velocity derived from the coherent radar measurements. Furthermore, Section 5 deals with improving the knowledge of the complex MTF by empirical and theoretical considerations. In Section 6 the developed MTF is applied, and the surface elevation reconstructed by the noncoherent part of the signal is compared to the surface reconstruction of the coherent part of the signal. The discussion in Section 7 mainly focuses on explaining the differences in the MTF developed herein to MTFs applied in previous works. Finally, Section 8 describes the conclusions of the work.

2. Radar data analysis

a. Radar specifications

The datasets used herein were recorded by a radar with an antenna mounted at height $H = 43$ m on the Research Platform Forschungsplattformen in Nord- und Ostsee Nr. 3 (FINO3; 55°11.7’N, 7°9.5’E) in the North Sea (see Fig. 1), located approximately 80 km west of the island of Sylt. The area has an approximately uniform water depth $h = 22$ m with tidal currents of up to $0.7$ m s$^{-1}$ (Huang et al. 2016). The radar operates under grazing incidence conditions with an incidence angle of $\theta = \tan^{-1}(r/H) > 77^\circ$, with the ground range $r$ (see Fig. 2). The radar (see Table 1) is coherent on receive; that is, the phase-resolved electromagnetic signal is measured when emitted and received. The receiver measures voltage at the two channels $(I$ and $Q$), which is proportional to the amplitude of the electromagnetic field. Therefore, in the text we denote the modulus of this signal provided by the receiver as “amplitude” to distinguish from the intensity, which is calculated as $I^2 + Q^2$. The backscattered signal is then composed of the amplitude of the received signal and the relative phase of the received signal, that is, the received phase minus the emitted phase. While the amplitude varies with the local incidence angle $\theta$, the relative phase depends on the Doppler frequency resulting from the surface velocity toward the radar $v_D$. The latter results from the projection of the surface velocity $v$ into the direction of $b$ pointing toward the radar (see Fig. 2a). It should be noted that for short-crested waves, or a radar look direction not aligned with the wave direction, the three angles $\theta$, $\theta_b$, and $\theta_c$ are not in the same plane.

The radar is operated in two modes: First, the rotating mode, recording temporal sequences of backscatter images. From this spatiotemporal information, the amplitude images are analyzed by the standard techniques derived for marine radars (Young et al. 1985; Nieto Borge and Guedes Soares 2000; Hessner et al. 2001). Hence, the peak wave direction is determined from the estimation of the directional wavenumber spectrum. Afterward, the radar operates in the fixed mode, where the radar antenna is pointing into the peak wave direction for 15 min. In this mode the radar measures the complex signal with the pulse repetition frequency of $1$ kHz and a pulse length of $50$ ns, resulting in the range resolution of $7.5$ m. The antenna is $2.3$ m long and is recorded with an azimuthal resolution of $1^\circ$.
The three-dimensional orbital motion of the waves is also measured by a directional heave–pitch–roll buoy in the area (55°11.2’N, 7°11.2’E); see Table 2.

**b. Doppler velocity measurements**

The phase-resolved backscatter signal is arranged into time intervals $\Delta t = 0.512$ s, and the phase differences $\Delta \chi_n$ between the phases of successive values are calculated within each time interval by

$$\Delta \chi_n = \chi_n - \chi_{n+1}. \tag{1}$$

The reliability of the phase difference measurement within each time interval is measured by calculating the alignment of the vectors associated with the phase shifts in the complex plane,

$$\text{Conf} = \frac{\sum_{n=1}^{N-1} e^{i\Delta \chi_n}}{N-1}. \tag{2}$$

A confidence close to one indicates a good agreement in the phase shift measurements, while a value closer to zero reveals disagreement in the estimated phase shifts.
TABLE 1. Technical details of the radar.

<table>
<thead>
<tr>
<th>Maker</th>
<th>GEM Elettronica</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>12-kW marine X-band radar</td>
</tr>
<tr>
<td>Coherence addition</td>
<td>University of Electronics,</td>
</tr>
<tr>
<td></td>
<td>St. Petersburg, Russia</td>
</tr>
<tr>
<td>Pulse repetition frequency</td>
<td>1 kHz</td>
</tr>
<tr>
<td>Pulse rate</td>
<td>9.48 GHz</td>
</tr>
<tr>
<td>Pulse length</td>
<td>50 ns</td>
</tr>
<tr>
<td>Range resolution</td>
<td>7.5 m</td>
</tr>
<tr>
<td>Maximum range</td>
<td>3262.5 m</td>
</tr>
<tr>
<td>Antenna size</td>
<td>2.3 m</td>
</tr>
<tr>
<td>Azimuth beamwidth</td>
<td>21°</td>
</tr>
<tr>
<td>Polarization</td>
<td>Vertical transmit and vertical</td>
</tr>
<tr>
<td></td>
<td>receive (VV)</td>
</tr>
</tbody>
</table>

TABLE 2. Technical details of the buoy.

<table>
<thead>
<tr>
<th>Maker</th>
<th>Datawell BV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>Datawell Mark III</td>
</tr>
<tr>
<td>Diameter</td>
<td>0.9 m</td>
</tr>
<tr>
<td>Mooring depth</td>
<td>25 m</td>
</tr>
<tr>
<td>Frequency resolution</td>
<td>1.28 Hz</td>
</tr>
</tbody>
</table>

\[
\nu_v = \sum_j \left( \omega_j - \mathbf{k}_j \cdot \mathbf{U} \right) \sinh [k (z + h)] \frac{\eta(k_j, \omega_j) e^{i(k_j \cdot r - \omega_j t)}}{\sinh (k h)}.
\]  

where \( \eta(k_j, \omega_j) \) are complex coefficients, \( r \) is the horizontal position vector relative to the radar, and \( t = |r| \) is the horizontal distance to the radar commonly known as ground range. Terms \( k, k_j = |k_j| \), and \( \omega_j = \omega(k_j) \) are the wavenumber vector, the wavenumber, and the angular frequency satisfying the dispersion relation

\[
\omega = \sqrt{g k \tanh (kh) + \mathbf{k} \cdot \mathbf{U}},
\]

respectively, where \( g \) is the acceleration of the gravity, \( \mathbf{U} \) is the current, and \( h \) is the water depth.

The three-dimensional discrete Fourier transform (DFT), \( \nu_D(k_n, \omega_m) \), of the Doppler velocity, \( \nu_D(r, t) \), results in complex coefficients, corresponding to a uniform grid \( (k_n, \omega_m) \) with \( N \) points in space and \( M \) points in time. The numbering of the discrete grid does not correspond to the index \( j \) in the theoretical Eqs. (4)–(6). The latter numbers refer to the coefficients of Fourier series and its related frequency–wavenumber pairs fulfilling the dispersion relation. The received signal may contain other contributions that do not correspond to waves, such as the spectral background noise. In addition, not all combinations of indices \((n, m)\) correspond to free waves; thus, we introduce a filter to extract the components associated with linear gravity waves. The spectral image of the acquired data shows the wave energy as a cloud around the theoretical dispersion relation in \((k, \omega)\). This cloud is called a dispersion shell and its width is denoted by \( \varepsilon \) (Young et al. 1985). The filter is defined by a pass-band filter around the dispersion relation so that the spectral components within the dispersion shell are selected,

\[
\Delta_{n,m} = \begin{cases} 
1, & \text{where } |\omega_m - \sqrt{g k_n \tanh (k_n h) - \mathbf{k}_n \cdot \mathbf{U}}| < \varepsilon \\
0, & \text{otherwise}
\end{cases}
\]  

The dispersion shell has to be wide enough to contain the energy contribution around the dispersion relation, given by Eq. (7), and narrow enough to exclude spectral background noise. This noise is due to the roughness of
the sea surface, which is responsible of the speckle noise in the radar (Nieto Borge et al. 2008). As the signal results from the roughness, modulated by the waves, the spectral background noise in the intensity modulation spectrum is important for the signal-to-noise ratio and the estimation of $H_s$, etc. (Nieto Borge et al. 2008). During the implementation, different filter widths have to be tested. It is expected that the wavenumber and frequency resolution (for our analysis $\Delta k = 0.013 \, \text{rad} \cdot \text{m}^{-1}$ and $\Delta \omega = 0.007 \, \text{rad} \cdot \text{s}^{-1}$, respectively)—and in the case of the fixed mode, the directionality of the wave field—play an important role when defining $\Delta_{n,m}$. In our case the dispersion filter was found by first identifying all pixels touching the dispersion curve. The pixel boundaries result from the discretization in wavenumber and frequency. An efficient algorithm for identifying the relevant pixels is to define a fine $k$ grid, calculate the corresponding $\omega$ values from the dispersion relation, and mark the pixel containing the $(k, \omega)$ value in the original wavenumber frequency grid. In the present study, the resolution in frequency is considerably higher than that in wavenumber; therefore, the resulting initial shell is wider where the dispersion curve is steeper and more narrow where the dispersion curve is flatter. For our case the initial shell is extended by $\pm \Delta k$. In practice, it appears that a filter that is slightly wider than the dispersion cloud does not change the results considerably.

Assuming that the Doppler velocity [Eq. (3)] is equal to the particle velocity [Eqs. (5) and (6)] at the surface, in the direction of $\mathbf{b}$, thus $\nu_T = \mathbf{b} \cdot \mathbf{v}$, and limiting consideration to those indices $(n, m)$ that are inside the filter [Eq. (8)], we need to solve $\hat{\nu}_D(k_n, \omega_m)$ from the equation

$$\hat{\nu}_D(k_n, \omega_m) \Delta_{n,m} = \mathcal{F} \{ \mathbf{b} \cdot \mathbf{v} \},$$

where $\mathcal{F}$ denotes the DFT with respect to space and time.

For the special case that $\mathbf{b}$ is constant over the radar footprint, we have

$$\hat{\nu}_D(k_n, \omega_m) \Delta_{n,m} = (\omega_m - k_n \cdot \mathbf{U}) \left( \frac{\mathbf{b} \cdot k_n}{k_n \tanh(k_n h)} - ib_z \right) \hat{\eta}_{D,n,m},$$

where $b_z$ is the vertical component of $\mathbf{b}$. Note that $b_z$ introduces an additional phase shift. This equation is ill-conditioned for simultaneous grazing incidence ($b_z \approx 0$) and wave directions orthogonal to the radar look direction ($\mathbf{b} \cdot \mathbf{k}_x \approx 0$). In the following we shall apply this equation under the assumption of grazing incidence and wave directions nearly parallel to the radar look direction, thus $b_z \approx 0$ and $\mathbf{b} \cdot \mathbf{k}_x \approx k_x$.

### Algorithm for the analysis

The Doppler images are processed according to algorithms widely used for amplitude images in operational settings (Young et al. 1985; Nieto Borge and Guedes Soares 2000; Hessner et al. 2001), with the additional step of mapping from the Doppler velocity to the surface elevation. The following gives a brief overview of the main steps from the initial data to the surface elevation [Eq. (10), where $b_z = 0$]. The crucial part of the standard procedure is to select the spectral components that can be associated with wave energy, in the current case limited to the contribution of linear waves. Here we will apply the algorithm to two-dimensional datasets resulting from the fixed mode; however, it is also valid for applications with the rotating antenna.

All steps are listed in chronological order:

1. Calculate the multidimensional DFT to transfer from the $(x, t)$ space to the $(k, \omega)$ space.
2. Apply a high-pass filter to avoid static and quasi-static patterns resulting from long-range radar imaging effects (Nieto Borge et al. 2004). Hence, the spectral values for low frequencies $\omega < \omega_{\text{threshold}}$ and low wavenumbers $k < k_{\text{threshold}}$ that cannot be considered in the range of wind-generated ocean gravity waves are suppressed. The term $\omega_{\text{threshold}}$ is the threshold of the high-pass filter; in this work $\omega_{\text{threshold}} = 2 \pi f_{\text{threshold}}$, where $f_{\text{threshold}} = 0.035 \, \text{Hz}$ and $k_{\text{threshold}}$ is calculated from $\omega_{\text{threshold}}$ by the dispersion relation.
3. Estimate the current $\mathbf{U}$ for defining the dispersion filter (Young et al. 1985; Senet et al. 2001):
   (i) Define a subset of data points $(G)$ exceeding a given spectral density threshold (20% of the spectral peak).
   (ii) Estimate $\mathbf{U}$ by minimizing the functional below for all chosen pixels,

$$F = \sum_{n \in G} \left[ \omega_n - \sqrt{gk_n \tanh(k_n h)} - b_z \mathbf{k}_n \cdot \mathbf{U} \right]^2 \quad (11)$$

Hence, the algorithm depends on a homogeneous water depth that must be known for shallow and moderate water depths.

(iii) Correct the estimate of the dispersion curve by taking into account the data points that are located close to the initial dispersion curve. In this second step the threshold for considering a data point is lower than in the first step with 2% of the spectral peak (Senet et al. 2001).
4. Apply the bandpass filter of Eq. (8) to the wavenumber and frequency components, obtained from the DFT output.
5) Map the Fourier coefficients of the Doppler velocity to the surface elevation as described in Eq. (10).
6) Invert the surface from the processed Fourier coefficients by applying an inverse multidimensional DFT (Nieto Borge et al. 2004).

4. Analysis of the Doppler signal in the fixed mode with grazing incidence

a. Implications of the fixed mode

For the rest of this paper, we consider the fixed mode with a nonrotating antenna having been oriented in the peak wave direction. We define the $x$ axis parallel with the radar look direction, implying $r = |x|$. One important implication of the fixed mode is that the dispersion shell, originally a three-dimensional cone, is projected into the $(k_x, \omega)$ plane. Generally, wave energy may therefore be located anywhere between the dispersion curves for positive and negative $k_x$. Only when the directionality of the wave field is sufficiently small does the pass-band filter around the dispersion curve for waves in the peak direction include all the wave energy.

b. The directional projection

In fixed mode the DFT will capture only the frequency $\omega_m$ and the $x$ component of the wavenumber vector $k_{xn}$. The $y$ component of the wavenumber vector and the wavenumber itself can then be estimated from the dispersion relation [Eq. (7)].

All wave components with some angle $\phi(k_{xn}, \omega_m) \neq 0, \pi$ between the wave direction and the radar look direction are imaged as projections in the radar look direction; see Fig. 3. These oblique waves lead to an underestimation of the horizontal velocity and thereby the surface elevation. Depending on the degree of directionality, the effect on the reconstructed surface elevation will be mild or strong.

c. The incidence projection

As mentioned above, we assume that the vertical velocity contribution is negligible in the velocity measurement. If we assume that all wave components in the wave field are approximately parallel with the radar look direction, we have

$$v_D \approx v_h.$$  \hspace{1cm} (12)

From Eq. (5) $v_D$ is expected to be in-phase with the surface displacement $\eta$. For long-crested waves, and with the radar look direction aligned with the wave direction, the correct relation between $v_D$ and $v_h$ can be calculated from geometric principles (see Fig. 2).

$$v_h = \frac{\sin(\theta_v) v_D}{\cos(\theta - \theta_v)}. \hspace{1cm} (13)$$

Figure 4 verifies Eq. (12) for a randomly generated unidirectional wave field with a distance of at least 200 m from the radar. The deviation manifests as a phase shift, as predicted by Eq. (10). As expected, the deviation is more pronounced close to the radar but, with a maximum deviation of 0.06 $H_s$ for the reconstructed surface, it is still insignificant.

d. Estimation of the wave elevation from the Doppler velocity

It remains to express $\hat{\eta}(k_{xn}, \omega_m)$ as a solution for Eq. (10) for fixed mode and grazing incidence. Before relating the measurement to a surface elevation, we look at the spectral images of the raw Doppler velocity (Fig. 5a) and the raw amplitude image (Fig. 5b). In both images the main energy is located on or to the right of the dispersion relation of waves in the $x$ direction. The indicated dispersion relation and the first harmonic include a Doppler shift caused by the current in the $x$ direction. Because of the different signal-to-noise ratios in the Doppler and amplitude images, the estimate of the current typically differs slightly. Minor differences in the current estimate do not affect the result. The dispersion filter is wide enough to tolerate small errors in the current estimate for the wavenumber segment relevant for the given resolution. From the definition of the dispersion relation it is apparent that low wavenumbers are less affected by the current than high wavenumbers.

The wave energy that lies to the right of the dispersion relation is due to waves propagating with some angle to the radar look direction such that $k > k_i$. In the case of
the Doppler velocity spectrum, some energy is also registered around the first harmonic and the subharmonic; in the literature it is often denoted as “group line” (Lund et al. 2014; Stevens et al. 1999). The energy along the dispersion relation is due to free gravity waves. For the first harmonic and the subharmonic, both imaging effects and nonlinear hydrodynamic effects play a role. A description of the first harmonic induced by the radar imaging effects can be found in Seemann et al. (1997) and Nieto Borge et al. (2008). Furthermore, the subharmonic is described in Frasier and McIntosh (1996) for radar measurements. A discussion of nonlinear hydrodynamic effects on the width of the dispersion shell were analyzed numerically (Krogstad and Trulsen 2010; Taklo et al. 2015) and experimentally (Taklo et al. 2015).

Figure 5 shows the \((k_x, \omega)\) spectrum, that is, the three-dimensional dispersion shell in \((k, \omega)\) projected into the \((k_x, \omega)\) domain. Only when the directional spread \((\sigma_1)\) is sufficiently small does the projected dispersion shell appear as a curve, allowing to formulate the dispersion filter by a pass-band filter around the dispersion curve for waves in the \(x\) direction. However, the directional projection, described in section 4b, manifests itself though the distribution of energy to the right of the dispersion curve. When \(\psi_{n,m} \approx 0\) and \(\tanh(k_x h) \approx \tanh(k_{n,m} h)\), Eq. (10) leads to the following relation
between the coefficients of the surface elevation and the Doppler velocity:

\[\hat{\eta}_D(k_x, \omega_m) = \frac{\omega_m^2 - k \cdot U}{k_x \cdot g} \hat{\rho}_D(k_x, \omega_m) \Delta_{n,m}.\] (14)

e. Comparing statistical parameters of the surface elevation from the Doppler signal and the buoy

The algorithm presented in the preceding section was applied to a data series of hourly records between 0944 local time (LT) 8 July and 0444 LT 10 July 2015. This period was characterized by a high significant wave height and a low directional spreading compared to the other datasets available. The high \(H_s\) implies both a reduced directionality and high-quality intensity images for proper comparison with the Doppler. The limitation of the directional spreading was desired in the interim stage of using data records of a fixed antenna. To minimize the effect of shadowing (<3%, based on Conf > 0.7), only near-range intervals from 200 to 700 m (\(\theta \in [78, 87]^{\circ}\)) were analyzed over the full time interval of 15 min.

Figures 6a and 6b document that \(H_s\) and \(T_p\), respectively, estimated from \(\hat{\eta}_D\) are in good agreement with the respective values of the nearby buoy. The significant wave height was calculated from the standard deviation \(\sigma\) of the reconstructed/measured sea surface elevation (\(H_s = 4\sigma\)). In the case of the Doppler data the spatiotemporal data were used, and in the case of the buoy temporal records over 20 min were used. For the peak period the centroid formula (IAHR–PIANC 1986) was applied,

\[T_p = 2\pi \frac{\int_{\omega_1}^{\omega_2} S(\omega) \, d\omega}{\int_{\omega_1}^{\omega_2} S(\omega) \omega \, d\omega},\] (15)

where spectrum \(S\), and \(\omega_1\) and \(\omega_2\) denote the first and the last points, respectively, around the spectral peak, where \(S(\omega) > 0.8 \max[S(\omega)]\). The errors between the statistical measures are given in terms of the normalized root-mean-square (NRMS) and the normalized bias (NB). For the significant wave height they are given as

\[
\text{NRMS} = \frac{\sqrt{\left[H_s(\text{radar}) - H_s(\text{buoy})\right]^2}}{\sqrt{H_s^2(\text{buoy})}} = 0.07, \\
\text{NB} = \frac{H_s(\text{radar}) - H_s(\text{buoy})}{H_s(\text{buoy})} = 0.01,
\] (16)

and for the peak period

\[
\text{NRMS} = \frac{\sqrt{(T_p(\text{radar}) - T_p(\text{buoy}))^2}}{\sqrt{T_p^2(\text{buoy})}} = 0.09, \\
\text{NB} = \frac{T_p(\text{radar}) - T_p(\text{buoy})}{T_p(\text{buoy})} = 0.01.
\] (17)

For more information about the \(H_s\) estimates from the Doppler velocity (with the given radar), the study by Carrasco et al. (2017a) should be consulted.
The presented sections show how the Doppler images of a coherent radar can be used to reconstruct the sea surface elevation and to retrieve statistical wave parameters that correspond well to those of a buoy. In many applications the available radar is noncoherent and the intensity signal has to be used. As mentioned above, the intensity images are not radiometrically calibrated and require interpretation. In the following sections we attempt to retrieve more information about the nature of the intensity signal as basis for an improved MTF.

5. Considerations for the MTF

a. Observed requirements for the MTF

There have been numerous attempts to translate the intensity image into a sea surface description by applying an MTF. Previously, the main focus was on defining the modulus of the MTF that results in reliable wave spectra. Early attempts by Wright et al. (1980), Feindt et al. (1986), and Schröter et al. (1986) focused on understanding the deviation between intensity spectra and Doppler velocity spectra resulting from coherent one-dimensional measurements in time. Later, Nieto Borge et al. (2004) found an empirical modulation transfer function by comparing the spectra of three-dimensional radar intensity images with spectra measured by a buoy. In the present work, the ambition is to revisit the problem of the MTF as a complex function. Unlike previous inversion techniques, where the phase shift caused by the imaging mechanism in the intensity data (Nieto Borge et al. 2004) was disregarded, both the phase and the modulus are incorporated.

The nature of the needed MTF is revealed by comparing the two datasets in the physical and spectral domain. Here, we assume that the Doppler signal and the intensity signal are independent measurements of the same wave field. The Doppler velocity is related to the velocity of the sea surface, while the intensity image results from the roughness of the sea surface modulated by gravity waves.

The spectral estimates in wavenumber of different input signals \( F(k) \) are compared in Fig. 7. We define the reference spectrum as the spectrum calculated from the surface elevation based on the Doppler signal. We observe that filtered spectra for intensity images of different range windows both deviate from the reference spectrum, with the far-range spectrum being closer to the reference spectrum than the near-range spectrum. By comparing the modulus of the given filtered spectra we conclude that the MTF must be a function of the wavenumber and the incidence angle.

In the next step, we focus on the phase shift between the amplitude image \( A \) and the Doppler image \( \nu_D \), \( \Delta \phi(k_x, \omega_m) = \phi_A(k_x, \omega_m) - \phi_D(k_x, \omega_m) \), resulting from the imaging mechanism. The simplest approach is to investigate the phase shifts between the Fourier transforms of the two images. Here we use the original data signals. For the sake of computational robustness, the phase shift is calculated from the cross-spectrum, \( \Gamma_{DA}(k_x, \omega_m) \). The latter is defined as the expectation \( E \) of the product of the Fourier transform of one signal,
\[ \hat{v}_D = |\hat{v}_D| e^{i\phi_D}, \]

with the complex conjugate of the Fourier transform of another signal, \( \hat{A}^* = |\hat{A}| e^{-i\phi_A} \):

\[
\Gamma_{D,A}(k_x,\omega_m) = \frac{1}{\Delta k_x \Delta \omega} \mathbb{E}\left[ \hat{v}_D(k_x,\omega_m) \hat{A}^*(k_x,\omega_m) \right] = \frac{1}{\Delta k_x \Delta \omega} \mathbb{E}\left[ |\hat{v}_D| |\hat{A}| e^{i(\phi_D - \phi_A)} \right], \tag{18}
\]

where \( \Delta k_x \) and \( \Delta \omega \) are the wavenumber resolution and angular frequency resolution, respectively. The argument (arg) of the cross-spectrum gives thus the phase shift between the Fourier components:

\[
\text{arg}\{\Gamma_{D,A}\} = \Delta \phi. \tag{19}
\]

Numerically, the expectation operator may be achieved by applying the cross-spectrum to subintervals and calculating the average of these intervals. Alternatively, the expectation may be performed by smoothing in the spectral domain. In the current example, the averaging was skipped. As the two signals are two measurements of the same sea surface, the phase shift between the two is expected to be deterministic and should not require averaging. A suitable filter may be based on discarding low-energy components as provided in Fig. 8c. The resulting Fig. 8 reveals an approximately constant angular shift in the dispersion shell, the subharmonic, and possibly the first harmonic (Senet et al. 2001). When considering only linear wave components, the phase shift is roughly 90° for the range \( r \in [200 \text{ m}, 700 \text{ m}] \).

As the imaging effect of the radar strongly depends on the range, the cross-spectrum gives only a partial answer with values averaged over the given range. Comparing the instantaneous phase of the signals at each point in range indicates a local phase shift \( \Delta \phi(r) \). The spectral phase difference and the instantaneous phase difference are thus complementary: The first defines the spectral dependence and the second the range dependence of the phase difference. The instantaneous phase is the argument of the analytic signal, a complex signal constructed from the original data and the Hilbert transform (\( \mathcal{H} \)) of that data (King 2009): The analytic signal of \( \eta \) is defined as

\[
\eta(t) + i \mathcal{H}[\eta(t)], \tag{20}
\]

where the Hilbert transform is defined as

\[
\mathcal{H}[\eta(t)] = \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} \frac{\eta(\chi)}{t-\chi} \, d\chi, \tag{21}
\]

with \( \mathcal{P} \) denoting Cauchy’s principal value.

Calculating the argument of the analytic signal is numerically unstable and therefore the \( \Delta \phi \) is calculated...
from the inner product of the analytic signal of $\eta_A$ and the analytic signal of $\eta_D$

$$\cos[\Delta\phi(t)] = \frac{\eta_D(t)\eta_A(t) + \mathcal{H}[\eta_D(t)]\mathcal{H}[\eta_A(t)]}{\sqrt{\eta_D^2(t) + \mathcal{H}^2[\eta_D(t)]}\sqrt{\eta_A^2(t) + \mathcal{H}^2[\eta_A(t)]}}.$$  

(22)

For the analysis of the instantaneous phase, datasets of 900 s in the range of 200–2500 m were used and the Hilbert transform was calculated for each point in space. Afterward, the phase differences were calculated according to Eq. (22) and averaged over time. Figure 9 shows the phase shift between the datasets ($\Delta\phi_1$) in comparison to phase shifts between a simulated sea surface and simple intensity models. For the latter the tilt modulation described in section 5b and the shadowing mask for the simulated sea surface were calculated. In $\Delta\phi_2$, the intensity model is combined from the tilt modulation and the shadowing mask, while the intensity model in $\Delta\phi_3$ is purely based on tilt modulation. From Fig. 9 we observe that the combination of tilt modulation and shadowing seems to be a good model for the intensity signal.

From the observations described above we conclude that the radar imaging process may be analyzed as a two-scale problem. The intensity signal changes slowly with the range and may be treated as locally range independent within short range intervals. However, the imaging mechanism is different when choosing different range intervals or when working under different incidence conditions. We define the MTF as a function of $k$ and the mean incidence angle over the interval, $\bar{\vartheta}$, and denote the MTF as $\mathcal{T}(k, \bar{\vartheta})$.

For simplicity, our MTF only redistributes the energy in the spectrum, while the scale correction $C$ of the spectrum is disregarded, as the radar is not calibrated. The term $C$ is usually determined to ensure that the estimated surface elevation gives the correct significant wave height. When working with a coherent radar, $C$ may be estimated from the Doppler signal; alternative techniques will be mentioned below. The Fourier transform of the surface elevation are retrieved from the Fourier transform of the filtered amplitude data $\hat{A}(k_x, \omega)$ by a multiplication with the MTF and the scaling constant,

$$\hat{\eta}_I(k_x, \omega) = C\hat{A}(k_x, \omega)\mathcal{T}(k_x, \bar{\vartheta}).$$  

(23)

b. Theoretical MTF for tilt modulation

To root the MTF in physical principles, the two scales, defined in the previous section, must be investigated separately. By choosing a small range window, the influence of the changing incidence angle is considered sufficiently small that it can be disregarded. From the analysis of the Hilbert transform, we anticipate that shadowing is the dominant effect for incidence angles approaching 90$^\circ$. As shadowing is a highly nonlinear phenomenon that depends on a large number of environmental influence factors and the available radar, we focus on incidence angles, where shadowing occurs only to an extremely limited degree (78$^\circ$ and 87$^\circ$). From the analysis in the previous section, we have reason to believe that the governing imaging mechanism for unshadowed regions is tilt modulation. We assume therefore that the amplitude image is related to the local
incidence angle $\theta_i = \theta_i(r, t)$ (cf. Nieto Borge et al. 2004). It is defined as the angle between the exterior normal to the sea surface $\mathbf{n} = (-\partial \eta/\partial x, -\partial \eta/\partial y, 1)$ and the orientation of the backscattered radar signal $\mathbf{b} = (-x, 0, H - \eta)/\sqrt{r^2 + (H - \eta)^2}$. The normal is calculated for a sea surface that is smooth on the scale of radar imaging mechanisms; that is, the small ripples that define the roughness of the sea surface are disregarded for the normal vector. The radar antenna is fixed and points into the sea surface parallel to the $x$ axis, resulting in $b_x = 0$. The angle between $\mathbf{n}$ and $\mathbf{b}$ can be calculated from the dot product,

$$\mathbf{n} \cdot \mathbf{b} = |\mathbf{n}| \cos(\theta_i). \quad (24)$$

We recall that $r = |x|$ and that the radar points along the positive $x$ axis ($x > 0$). Further, using $\eta \ll r$, $H \ll r$, and $(\partial \eta/\partial x)^2 + (\partial \eta/\partial y)^2 \ll 1$, the equation above can be simplified to

$$\frac{r}{H - \eta} \frac{\partial \eta}{\partial x} + (H - \eta) = \sqrt{1 + \left(\frac{\partial \eta}{\partial x}\right)^2 + \left(\frac{\partial \eta}{\partial y}\right)^2} \cos(\theta_i), \quad (25)$$

resulting in

$$\frac{\partial \eta}{\partial x} + \frac{H - \eta}{r} = \cos(\theta_i). \quad (26)$$

The incidence angle is related to the amplitude $A(x, t)$ by the radar equation (Skolnik 2000). Since the radar used herein is not calibrated, the constants that affect the received power are unknown. Furthermore, we disregard the power decay, as it is assumed that the decay has previously been compensated in the amplitude data. Therefore, we consider only the normalized radar cross section $\sigma^0$ in the radar equation, where $\sigma^0$ is proportional to the received power. As a result, the range-corrected amplitude signal is proportional to the square root of the normalized radar cross section,

$$A \propto \sqrt{P_r} \propto \sqrt{\sigma^0}. \quad (27)$$

Following the approach by Rees (2013, 282–284), the normalized radar cross section may be approximated by the following relation to the local incidence angle:

$$\sigma^0 \propto \cos^2(\theta_i), \quad (28)$$

resulting in

$$A \propto \cos(\theta_i). \quad (29)$$

It should be noted that Eq. (28) is not well confirmed for extreme grazing incidence conditions and that its validity may depend on the radar, the polarization, and possibly other influence factors.

Finally, we can apply $\mathcal{T}$ to Eq. (26), where we replace $\cos(\theta_i)$ with $CA$,

$$\mathcal{T}\left(\frac{\partial \eta}{\partial x} + \frac{H - \eta}{r}\right) = C \mathcal{T}\{A\}, \quad (30)$$

where $C$ is a constant that depends on a number of factors. The constant may be defined by calibration or by comparison to other measurements.

By further simplification based on

$$|\mathcal{T}\left(\frac{\partial \eta}{\partial x}\right)| \gg |\mathcal{T}\left(\frac{-\eta}{r}\right)| \quad (31)$$

and

$$\mathcal{T}\left(\frac{H}{r}\right)(k) = 0, \quad \forall k > k_{\text{threshold}} \quad (32)$$

reduces the tilt MTF to

$$ik \mathcal{T}\{\eta\} = C \mathcal{T}\{A\}, \quad (33)$$

As mentioned above, we are interested in understanding the relation between the spectral components of the amplitude image and the wave elevation. The overall scale of the amplitude image is assumed to require a highly nonlinear description of multiple influence factors, which is outside the current scope. The relation between surface elevation and amplitude image for data with little shadowing is given by

$$\tilde{\eta} = -ik^{-1} CA, \quad (34)$$

leading to

$$\mathcal{T}\{k\} = -ik^{-1} C. \quad (35)$$

The given tilt transfer function is a simplified version of Alpers and Hasselmann (1978) by defining the radar cross section as given in Eq. (28). The MTF implies a phase shift by $90^\circ$ of all spectral components and a division of all components $I(k_n, \omega_m)$ by their respective wavenumber $k_n$. Consequently, the amplitude of the backscatter measures the slope of the waves such that the slope facing the radar triggers the maximum return of the signal and the slope behind the crest returns the least.
Applying the developed MTF to the amplitude spectrum shows good agreement with the spectrum derived from the Doppler signal for image sections from near range (see Fig. 10). It should however be mentioned that contributions with $k < 0.03 \text{ rad m}^{-1}$ were more similar without the MTF. Note also that we apply the average phase shift between the signals derived from the analytic signal instead of 90°. For all the cases analyzed within this work, the difference was marginal and the value of 90° may be used in case of a noncoherent radar.

6. Comparison of measurements

In the following, the data analyses of amplitude and Doppler images are presented together. The datasets and range windows used for the amplitude images were the same as those described in section 4e, and the algorithm presented in section 3 was adapted for the amplitude analysis by three additional steps: Before step 1, the range-dependent decay in the amplitude data resulting from the radar equation (Rees 2013, 282–284) is removed. Since the near-range examples are characterized by almost no shadowing, this is achieved by dividing each range bin of the amplitude data by its mean value in time. Between steps 3 and 4 of the Doppler algorithm, the MTF is applied, in order to pass from the Fourier transform of the amplitude signal to the unscaled Fourier transform of the estimated surface elevation (Young et al. 1985). And finally, the wave height of the amplitude image is scaled before or after step 6. In the case of wind sea, the signal-to-noise ratio may be used instead (Ziemer and Günther 1994; Nieto Borge 1998); other attempts for estimations of $H_s$ are based on the shadowing (Wetzel 1990) or other sea-state parameters, such as the wave period (Gangeskar 2014). In the present work, the amplitude image is scaled to match the significant wave height of the Doppler image.

The surface reconstructions coincide generally well. The extracted representations are the surface along the entire range for the time step $t = 410 \text{ s}$ (see Fig. 11a) and the time evolution for the surface in the range position $r = 225 \text{ m}$ [Eq. (11b)]. For both extractions, the associated root-mean-square error between the inversion from the Doppler and the inversion from the amplitude is indicated [RMS($t$) and RMS($r$), respectively]. Because of the limited resolution, the signals were interpolated by a truncated Fourier series expansion. As documented in Fig. 12, the two surface inversions along the range at $t = 410$ agree well for all cases.

Further insight into the imaging mechanism is gained by comparing the estimated surface elevations with the original signals for near range (Fig. 13a) and for far range (Fig. 13b). As expected, we see clear differences between the raw amplitude and the Doppler signal, but after the surface reconstruction, the two signals show a strong resemblance. The confidence [defined in Eq. (2)] drops behind each crest. The effect is minimal in the near range and more pronounced farther away from the radar. These observations support the assumption of the confidence being an indicator for shadowing. The nature of shadowing has, among others, been investigated by Plant and Farquharson (2012) and is distinguished between geometric shadowing and partial shadowing. Geometric shadowing is assumed as the

![Fig. 10. Comparison of normalized wavenumber spectra $F(k)$ generated from filtered Doppler and amplitude data with the MTF applied.](image-url)
most conservative approach for defining shadowed areas. It classifies areas as shadowed when a straight line from the radar to the area crosses another wave. According to Plant and Farquharson (2012), partial shadowing is characterized by diffracted radar backscatter from the areas that are classified as geometrically shadowed. In Fig. 13b the shadowing in the intensity signal may be well approximated by geometric shadowing. Behind the wave crests, where the confidence drops, the amplitude remains at a constant low level. In contrast to that, the influence on the Doppler signal is less pronounced except for certain cases, characterized by spikes that are unlikely to correspond to surface waves. When the dispersion filter has been applied, the spikes forming the raw signal disappear and the reconstructed surface is smooth.

Since the buoy is located outside the radar footprint, a direct comparison of the wave elevation was impossible. An additional validation of the radar by means of the buoy is a spectral comparison. Figure 14 shows a typical case with good resemblance. When there are two peaks, the radar usually underestimates one of them. In general, energy with $k > 0.2 \text{ rad m}^{-1}$, corresponding to $\omega > 1.4 \text{ rad s}^{-1}, T_p < 4.5 \text{s}$, is not well resolved because of the poor spatial resolution and dispersion filtering.

Finally, we investigate the influence of the directional spreading to the reconstructed surface. To estimate the surface reconstruction error resulting from the directional projection, the buoy data were consulted. A synthesized Doppler velocity is composed from the time derivatives of the horizontal positions (east and north) processed according to Eq. (14). The RMS error between the surface elevation measured by the buoy and the one synthesized from the horizontal buoy displacement is around 10%, increasing with the directional spreading (Fig. 15a). Similarly, the RMS between the Doppler and the amplitude surface reconstruction increases with the directional spread, when the MTF is applied; see Fig. 15b. Not applying the MTF (except for the phase shift correction) leads to a higher RMS that decays with increasing directional spreading.

7. Discussion
a. The influence of shadowing on the MTF

In the previous section we have found that the proposed tilt-based MTF seems to fit well in our case. In contrast to many previous publications, where the MTF was applied to the spectral estimate in the $k$ domain (e.g., Nieto Borge et al. 2004), our MTF is applied to the Fourier components of the estimated surface elevation $\eta_A$ in the $(\mathbf{k}, \omega)$ domain. We use the exponent $\alpha$ for the Fourier components and $\beta = 2\alpha$ for the corresponding values of the power spectrum,

$$|F(k)|^2 = k^{-\beta}.$$  (36)

In the following, the developed tilt MTF with $\beta = 2$ is put into relation to the often-used empirical MTF with $\beta = 1.2$, introduced by Nieto Borge et al. (2004).
FIG. 12. Inversion comparisons (analog to Fig. 11a) for all cases.
Their MTF was estimated from averaging over a number of spectra from different recordings in the Bay of Biscay, a swell-dominated area, with a rather low significant steepness \( H_s = gT_p^2 \) and significant wave heights ranging from 1.9 to 4.7 m. The most striking difference with the current case is the high incidence angle \( \theta \approx 88^\circ \), which is associated with a considerable amount of shadowing. Similar to the range dependence shown in Fig. 7, Lund et al. (2014) found that the observed spectrum changes with the range, as documented in Fig. 20a of Lund et al. (2014). The figure shows estimated wave spectra for near range (200–700 m), midrange (700–1200 m), and far range (1700–2200 m), where Lund et al. have applied the same MTF (Nieto Borge et al. 2004) to all. Lund et al.’s best result in accordance with the buoy spectrum is given

![Graph showing spectral comparison with the buoy](image1)

**Fig. 13.** Qualitative comparison of radar measurements and the estimated surface elevation for (a) near range and (b) far range. Terms \( D \) and \( A \) denote the raw signals of Doppler and amplitude, respectively, with the corresponding surface reconstructions \( \eta_D \) and \( \eta_A \), which are identical to the ones in Fig. 11. The confidence (Conf) is defined in Eq. (2).

![Graph showing spectral comparison](image2)

**Fig. 14.** Spectral comparison with the buoy. The logarithmic plot includes power decay laws and the angular cutoff frequency for the radar.
for the midrange, which had a similar incidence angle as the one used by Nieto Borge et al. (2004). Compared to the midrange spectrum, the near-range spectrum has more energy in the high-frequency tail and less in the peak, while the far-range spectrum is characterized by more energy in the peak and less in the tail. As described above, we assume that the increasing shadowing with the increasing incidence angle requires an MTF with a lower exponent. Therefore, we expect that applying the correct MTF corresponding to the amount of shadowing in the given range intervals may have resulted in similar spectra for all ranges.

In the following we test the possible influence of shadowing on the MTF by a mathematical model. We use the simple tilt modulation model defined in section 5b and modify it by a shadowing mask. The areas that are shadowed are defined from geometric shadowing and the values are chosen according to the signal-to-noise ratio applicable for the given incidence angle; that is, close to the radar, the shadowed areas have a relatively lower value compared to the illuminated areas, and farther away the intensity values of shadowed areas are only slightly below the illuminated areas.

Figure 16a shows the range-dependent behavior of an intensity model purely based on tilt modulation and shadowing. In accordance with Fig. 7, the model based on tilt and shadowing captures how the increasing incidence angle modifies the decay of the wavenumber spectrum, while the tilt alone (Fig. 16b) results in practically identical spectra for all incidence angles. Different models for the shadowing mask were tested by imposing alternative values in shadowed areas, but the effect of these changes on the received spectra were

![Figure 15](image15.png)

**FIG. 15.** Reconstruction error (RMS/Hs) between surface reconstructions from (a) the buoy and (b) the radar drawn over the corresponding value of directional spreading calculated according to Kuik et al. (1988).

![Figure 16](image16.png)

**FIG. 16.** Spectral estimates from a wave field and simulated intensity images based on (a) tilt modulation and shadowing and (b) tilt only. The different intensity images correspond to three different incidence angles.
minimal. Thus, the governing effect lies in the ratio between the shadowed and unshadowed areas.

The findings above show that the shadowing has approximately the effect of multiplying the filtered spectrum by $k^d$ and thereby reducing the power of the MTF. The value of $d$ depends on the amount of shadowing present. We conclude that the MTF may be approximated by the multiplication of a tilt and a shadowing contribution, $\mathcal{S} = \mathcal{S}_{\text{tilt}} \cdot \mathcal{S}_{\text{shadow}}$. It should, however, be clarified that $T_{\text{tilt}}$ is an MTF that is calculated from physical effects and therefore is expected to be very accurate under grazing incidence conditions, while $\mathcal{S}_{\text{shadow}}$ is an empirical estimate of an advanced mechanism approximated by a power law of $k$. Therefore, we suggest avoiding shadowed data whenever possible. If shadowing is unavoidable because of local incidence conditions, it is not advisable to use a generic MTF.

b. Applicability of the MTF

It has previously been described that the MTF depends on a large number of factors, ranging from environmental conditions, such as wind speed and direction, to radar-specific parameters, including radar wavelength, incidence angle, and look direction (Hwang et al. 2010).

To minimize the number of influence factors on the imaging mechanism of the radar, the following strategy may be advisable: The chosen range interval should be small to limit the effect of the increasing incidence angle (e.g., five peak wavelengths) and it should be as close as possible to the radar antenna to limit the effect of shadowing. In addition, a minimum threshold for $H_s$ should be chosen in order to minimize the influence of the wind. According to Hessner et al. (2001), reliable wave measurements with X-band radar with horizontal transmit and horizontal receive (HH) polarization are obtained in the presence of a minimum wind speed is necessary (typically, higher than $3 \text{ m s}^{-1}$). With the coherent radar at FINO3 used in this study, $H_s$ could be well estimated merely based on Doppler images of $H_s > 0.5$ m and on intensity images of $H_s > 1.0$ m (Carrasco et al. 2017b). The most appropriate range window was found to be defined by the incidence angles between $78^\circ$ and $87^\circ$.

8. Conclusions

The presented comparison of Doppler and amplitude images sheds light on the imaging mechanisms of a noncoherent radar. Though influenced by the same parameters, the Doppler and intensity images may be considered as independent measurements of the sea surface. As a result of analyzing both the Doppler and intensity images and comparing the results, the following conclusions may be drawn.

The inversion technique applied to the Doppler images results in a sea surface reconstruction with $H_s$ and $T_p$ estimates comparable to that of a buoy. For applications of a noncoherent radar under grazing incidence conditions with limited shadowing, tilt modulation was found to be the governing imaging mechanism. The resulting MTF

$$\mathcal{S}(k) = ik^{-1}C$$

was derived from relation between the local incidence angle and the radar cross section under the assumption of grazing incidence.

The quality of the analysis depends highly on the choice of the datasets. A suitable range window has to be chosen and a threshold for significant wave height should be defined. A range window with the incidence angle of $\theta \in [78^\circ, 87^\circ]$ gave the best results in the present case, as it avoids shadowed regions, and the grazing incidence assumption is still fulfilled and the window is wide enough. In the future, the choice of the range window is expected to allow various degrees of shadowing, based on a technique to deal with shadowed regions that is currently under development.

Finally, sea surface inversions from the Doppler and amplitude are consistent in our study. In combination with good agreement between the statistical parameters of the Doppler signal and the buoy, the results are trustworthy for observations with limited directional spreading and medium-to-high significant wave heights. The presented solution with the Doppler data is superior to the amplitude-based solution, since the MTF and the scaling of the surface elevation are redundant. However, with a coherent radar, it may be advantageous to use both datasets for cross validation of the results.

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