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2 **Impact of atmospheric stability on X-band and C-band Synthetic Aperture Radar**
3 **imagery of offshore windpark wakes**

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9 C-band and X-band Synthetic Aperture Radar (SAR) data acquired by the Sentinel-
10 1 and TerraSAR-X satellites are used to study atmospheric wakes behind offshore
11 wind parks in the German Bight. A particular focus is on the impact of atmospheric
12 stability on wake parameters like the wake length. Stability parameters are estimated
13 from measurements taken at the FINO-1 observation platform. Based on a data set
14 covering different seasons and concentrating on the first German offshore wind park
15 Alpha Ventus (AV), it is shown that in this area stable atmospheric conditions favour
16 longer wakes. This is first demonstrated for situations, where the wake behind AV
17 was unperturbed by other neighbor wind parks. In this case wakes of more than 30
18 km length are observed. In a second step the more complicated situation with wake
19 superposition from different neighboring wind parks is analysed. It is shown that in
20 this case the merged wakes can extend to more than 70 km downstream.

21 The analysis is challenged by two factors. First of all, the FINO-1 platform is
22 within the wind farm wakes for a certain range of wind directions. This means
23 stability estimates for the upstream conditions are not straightforward to obtain in
24 these conditions. The second complication is associated with an apparent increase
25 of radar cross section downstream of wind parks observed on many SAR scenes,
26 typically within the first 10 km downstream the wind park. A semi-empirical model
27 is proposed to explain this effect by an increased downward momentum flux associated
28 with increased turbulence generated by the wind park. Applying numerical inversion
29 methods, a couple of typical downstream wind speed profiles are reproduced with
30 this model based on SAR derived estimates of the friction velocity.

31 Keywords: Offshore windfarms, Synthetic Aperture Radar (SAR), Atmospheric sta-
32 bility

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I. INTRODUCTION

The wind energy sector has grown rapidly with a global 25% increase each year within the first decade of this century. Projections by the International Renewable Energy Agency (IRENA) estimate that the global use of wind energy will grow by 500% between 2010 and 2030¹. The wind sector contribution to renewable energy is expected to increase from 2% to 11% during that period. The wind energy production in Germany has reached about 13% of the electricity consumption in 2016. Due to higher mean wind speeds, lower turbulence intensity, as well as the increasing shortage of suitable locations on land, offshore wind farming has become an important factor as a renewable energy source². In 2016 the offshore wind parks (OWPs) installed along the German coast have produced 12.4 GWh, which corresponds to 2.1 % of the electric energy consumption in Germany. By the end of 2016, a total of 947 turbines have been installed and plans for more wind farms exist. Fig. 1 shows some of the major existing and operational OWPs in the German Bight by the end of June 2016.

The growing use of offshore wind resources has led to increased need for information on coastal zone conditions and possible environmental impacts of wind park installations. One important topic in this context is the generation of wakes in the atmosphere, which are potential factors for the operation of neighboring wind farms downstream. In this study such wake effects are investigated using a combination of satellite radar data and in-situ observations.

The extraction of energy and momentum from the wind field by offshore turbines can lead to pronounced wakes downstream. The wake region is generally associated with a velocity deficit, a pressure change and increased turbulence. The length of these wakes can be several tens of kilometers^{3,4} and should therefore be considered in the positioning of new installations downstream. The intensity and the extension of wakes is known to depend on different parameters like the atmospheric stability, wind speed as well as wind park size and turbine spacing^{3,5,6}. Different observation techniques have been used to study wakes behind wind turbine, e.g., in-situ instruments installed on fixed platforms or wind turbine nacelles (e.g., wind scanner lidars). Measurements from fixed platforms usually provide very good temporal sampling, but lack spatial coverage needed to analyse wake effects on a larger scale. On the other hand, Synthetic Aperture Radar (SAR) systems as flown on the

64 European SENTINEL-1 or the German TerraSAR-X satellite provide large spatial coverage
65 (up to several hundred kilometers) and high spatial resolution (down to a few meters), but
66 relatively coarse temporal sampling. SAR retrieval of wind information is based on the
67 correlation between the radar cross section and the small cm-scale sea surface roughness.

68 The overall objective of the study is to improve the interpretation of satellite SAR scenes
69 and to optimise the information extraction concerning offshore wind farm related param-
70 eters. The use of SAR for wind speed retrieval is well established and there is a large
71 number of publications about this subject including the radar frequency and polarisation
72 dependence⁷⁻⁹. The potential of SAR data to provide information on wind farm wakes has
73 already been demonstrated in previous studies^{3-5,8,10}. The first studies on this subject were
74 mainly concerned with the derivation of general wind statistics. These investigations are,
75 for example, related to the optimal siting of offshore wind farms^{11,12}. Recently, studies have
76 looked in more detail into the structure and dynamics of offshore wind farm wakes making
77 use of the high spatial resolution of SAR data⁴. First studies were also done considering the
78 impact of atmospheric stability on SAR observed wakes³.

79 In this study a couple of new aspects are addressed, which are important for the use of
80 SAR data in the context of offshore wind farm planning and operation, in particular in view
81 of the growing number of installations operating in closer proximity:

- 82 • A combination of X-band and C-band SAR data is used for the analysis of offshore
83 wind farm wakes. This approach increases the data basis and adds flexibility compared
84 to the sole use of C-band data in previous studies.
- 85 • The dependence of wake parameters on atmospheric stability is investigated for the
86 first operational wind park Alpha Ventus using SAR data in combination with FINO-1
87 platform measurements and numerical model data. Different parameters quantifying
88 the contribution of thermal stability and wind shear are considered. As a new com-
89 ponent an empirical model is fitted relating wake length to stability.
- 90 • The superposition of wakes from different wind farms is investigated by analysis of
91 SAR data acquired over Alpha Ventus and the neighbouring Trianel wind park. The
92 data set contains scenes both before and after the Trianel wind farm was installed.
- 93 • A semi-empirical model is proposed to explain the increase of radar cross section within

94 a region of about 10 km downstream of the turbine. The dominant factor in the model
 95 is an increased downward momentum flux due to mechanically generated turbulence.
 96 The model is required to avoid the misinterpretation of these radar cross section values
 97 as increased wind speeds at hub height, which would not make physical sense. This
 98 problem, which occurs when applying standard SAR wind speed retrieval schemes to
 99 imagery of offshore wind farms, is to our knowledge discussed for the first time.

100 The manuscript is structured as follows: Section II summarises the basic properties of
 101 offshore wind farm wakes and the existing research as well as a short overview of electro-
 102 magnetic wave scattering from the ocean surface in the microwave regime. The study area
 103 and the used data sets are introduced in Section III. The method applied to estimate near
 104 surface wind speed from SAR data is described in Section IV. In Section V the results on
 105 the impact of stability on wake length and velocity deficit are presented. A semi-empirical
 106 model, which is able to re-produce the radar cross section increase immediately behind the
 107 wind park is introduced in Section VI.

108 II. BASICS ON WIND FARM WAKES AND RADAR SCATTERING

109 A. Offshore Wind farm Wakes

110 In this study radar images like shown in Fig. 5a,b and Fig. 6a,b are used to analyse wakes
 111 downstream of offshore wind farms. The images show wakes of different length behind the
 112 wind farm Alpha Ventus (see Fig. 1). In the following we will discuss the interpretation of
 113 these radar images and approaches to retrieve useful information about different parameters
 114 and processes.

115 The analysis of wakes behind offshore wind farms has to take into account the particular
 116 conditions in the atmospheric boundary layer above the water. Today's offshore wind tur-
 117 bines have typical hub heights of about 100 m and rotor diameters of about 120 m length.
 118 These dimensions lead to significant interactions between the turbines and the boundary
 119 layer on different spatial and temporal scales.

120 The understanding of the respective processes is a challenge, because the dynamics is
 121 strongly affected by turbulence, which is notoriously difficult to handle both in theory and
 122 numerics. The turbulence created by offshore wind farms has typical length scales of 0.5

123 km^{3,13,14}. The situation over water is further complicated by the fact that there is a two-
124 way interaction mechanism between the surface roughness and the wind related friction
125 velocities.

126 The basic reason for the occurrence of wakes is the removal of momentum by the turbines.
127 The amount of momentum taken out of the boundary layer is depending on the wind speed
128 and the thrust coefficient of the respective turbine¹⁴. In the first place this leads to an
129 area of reduced wind speed in the downstream area behind the turbines. This reduction
130 then causes wind shear and hence turbulence at the boundary of the wake. The resulting
131 momentum mixing leads to a downstream spread and weakening of the wakes until it finally
132 disappears. The dispersion in the vertical in particular causes the wake to hit the water
133 surface at some distance downstream the turbine. This distance is typically of the order of
134 about 10 rotor diameters^{3,15}. The wake dispersion is naturally stronger if there already is a
135 lot of turbulence in the upstream boundary layer. For this reason wakes have a tendency to
136 be more pronounced under thermally stable atmospheric conditions¹⁶. Another consequence
137 is that wake effects are usually less intense at higher wind speeds (>10 m/s), which are
138 often associated with a higher shear production of turbulence. Exceptions are inversion
139 conditions, where the thermal stability is so strong that high wind speeds can occur with
140 relatively small turbulence intensities.

141 Wakes have been described by models of different complexity. A simpler 1D model,
142 which basically describes the momentum budget under different stability conditions was
143 proposed by Emeis¹⁶. Here, momentum is provided from above the turbines and both the
144 turbines and the sea surface act as momentum sinks. Frandsen *et al.*¹⁵ proposed a parametric
145 model based on the momentum equation, which tries to simulate the spatial structure of
146 wakes including the interaction of wakes from different turbines. Wakes of neighbouring
147 turbines typically merge after about 30 rotor diameters. More complicated 3D simulations
148 are usually done with numerical Large Eddy Simulation (LES) models. One example can
149 be found in Yang, Meneveau, and Shen¹⁷, where also the ocean response is treated in more
150 detail. One problematic issue in the LES models still seems to be the proper representation
151 of turbulence generation, dissipation and advection. Also the level of detail required to
152 describe the interaction of the rotating blades with the airflow is still subject of ongoing
153 research.

154 Further studies exist on particular aspects of wind turbine wakes, e.g., wake meandering

155 and survey¹⁸, or the impact on the local and global meteorology^{3,19,20}.

156 B. Microwave radar scattering from the sea surface

157 SAR systems are active radar instruments, which transmit and receive signals in the
 158 microwave frequency band. They are thus independent of daylight and are rarely affected
 159 by atmospheric conditions (e.g., extreme rain). The measurement of near surface wind
 160 speeds from SAR data is based on the so called Bragg scattering mechanism. For a facet
 161 tilted by an angle ψ in the x direction and by an angle δ in the y direction the radar cross
 162 section associated with Bragg scattering is given by²¹:

$$163 \quad \sigma_{Bragg}(\theta, \psi, \delta) = 16\pi^4 \cot^4(\theta_i) |\alpha(\theta, \psi, \delta)| \\ 164 \quad \times \Psi(2k \sin(\theta + \psi), 2k \cos(\theta + \psi) \sin(\delta)) \quad (1)$$

165 with incidence angle θ , local incidence angle $\theta_i = \cos(\theta + \psi) \cos(\delta)$, electromagnetic wavenum-
 166 ber k , and small scale wave spectrum Ψ . The radar wavelengths for the systems considered
 167 here are in X-band (2.5-4 cm) and C-band (4-8 cm). The plane of incidence is in x direction
 168 and α is a complex valued function of incidence angle, radar frequency, polarisation and
 169 dielectric properties of water. For a surface with various facets tilted in different ways the
 170 normalised radar cross section (NRCS) can be estimated as

$$171 \quad \sigma^0 = \int \frac{\sigma_{Bragg}(\theta, \psi, \delta)}{\cos \psi \cos \delta} p(\tan \psi, \tan \delta) d(\tan \delta) d(\tan \psi), \quad (2)$$

172 where $p(\tan \psi, \tan \delta)$ is the slope probability density function. This function can be derived
 173 from the ocean wave spectrum Ψ .

174 Based on dimensional considerations it can be shown that the high frequency ocean wave
 175 spectrum appearing in eq. 1 is of the following form

$$176 \quad \Psi(k, \phi) = k^{-4} f(\phi, ku_*^2/g), \quad (3)$$

177 where f is an unknown function²², u_* is the friction velocity and g is the acceleration of
 178 gravity. Eqs. 1-3 show that there is a direct connection between the normalised radar cross
 179 section σ^0 and the friction velocity u_* for a given wind direction. Making assumptions about
 180 the shape of the short wave spectrum, analytical models can be derived for the simulation
 181 of σ^0 from given wind conditions. Because these models still contain a lot of uncertain

182 components, empirical models are often preferred for practical applications like wind speed
 183 retrieval from radar. A recent comparison of physically-based and empirical models can be
 184 found in Fois²³.

185 As empirical models already exist for C-band and X-band, which are suitable for the
 186 incidence angle regimes of Sentinel-1 and TerraSAR-X data, we will follow this approach
 187 here. More details on how these functions were used for the wind speed retrieval in this
 188 study are given in Section IV A.

189 III. STUDY AREA AND USED DATA SETS

190 A. The offshore wind park Alpha Ventus

191 The present study is focused on the offshore wind park Alpha Ventus (AV). Starting
 192 operational production in April 2010, AV was the first offshore wind park at the German
 193 Coast. AV is located approximately 45 km north of the island Borkum and about 400 m
 194 east of the observation platform FINO-1 (see Fig. 1). More details on this platform will be
 195 given in Section III C. The offshore wind park consists of twelve 5-megawatt turbines, which
 196 are arranged in 3 columns and 4 rows with a spacing between 761 m and 849 m. The wind
 197 turbines have hub heights of about 90 m and rotor diameters of about 120 m. The cut-in
 198 wind speed of the turbines is ~ 3.5 m/s, while the rated wind speed is 13 m/s. At strong
 199 wind speeds between 25-30 m/s, the turbines are shut down for safety reason. AV covers an
 200 area of about 4 km² and is surrounded by neighbouring OWPs, such as Borkum Riffgrund
 201 installed in 2014 (~ 2 -4 km to the South) and Trianel installed in June 2014 (~ 5 -10 km to
 202 the west) (see the zoom in area in Fig. 1).

203 B. TerraSAR-X and Sentinel-1 data sets

204 TerraSAR-X was launched on 15 June 2007²⁴. It flies at a sunsynchronous orbit at 514
 205 km altitude with an exact repeat cycle of 11 days. It has three acquisition modes (Spotlight,
 206 StripMap and ScanSAR) with different polarisations such as VV, HH, VH, HV depending
 207 on the image mode. The StripMap and ScanSAR data used in this study have a spatial
 208 resolution of about 3 m and 18 m respectively and the across track image width is about 30
 209 km for StripMap and 100 km for ScanSAR.

210 The ESA SAR data used in this study were acquired by the Sentinel-1A satellite launched
 211 on 3 April 2014²⁵. The satellite also flies on a sunsynchronous orbit at about 800 km altitude.
 212 The exact repeat cycle is 12 days. The instrument has four acquisition modes (Strip Map,
 213 Interferometric Wide Swath, Extra-Wide Swath and Wave-Mode). The data used in this
 214 study were acquired in Interferometric Wide Swath mode, which provides images with across
 215 track width of about 250 km. The high resolution products provided in this mode have a
 216 resolution of about 20 m. This instrument also gives four possibilities for polarisation (VV,
 217 HH, VH, HV). In our study, vertical polarization for transmission and reception (i.e., VV)
 218 is used. VV is traditionally preferred for SAR wind speed retrieval, because of the higher
 219 NRCS (see eq. 2) obtained with this polarisation for the sea surface. It should be noted
 220 that additional SAR products exist that could potentially improve wind speed estimates. For
 221 example, TerraSAR-X has a fully polarimetric mode and SENTINEL-1 provides additional
 222 maps of Doppler centroid estimates, that can be linked to near surface wind speeds^{26,27}. The
 223 respective methods and theories are however less consolidated than the techniques used in
 224 this study and were therefore left for consideration in future studies.

225 Due to the sunsynchronous orbits of both satellites, data are always taken around the
 226 same time of the day. For the German Bight the TS-X and Sentinel-1A acquisitions are
 227 around 05:00 UTC and around 17:00 UTC.

Satellite	Band	Frequency [Hz]	wavelength [m]	Rep. Cyc. [days]	Resol. [mxm]
TerraSAR-X	X	9.6 GHz	0.031 m	11	3 m × 3 m 18.5 m × 18.5 m
S1A	C	5.4 GHz	0.055 m	12	20 m × 22 m

TABLE I. Some relevant imaging parameters for TerraSAR-X and S1A.

228 C. FINO-1 data

229 FINO-1 is a research platform installed at 54°N, 6°E 35' in the North Sea²⁸, which is about
 230 400 m west of the Alpha Ventus offshore wind park. It is devoted to measure meteorological
 231 and oceanographical parameter for research purposes. Air temperature is measured at 30

Dates [yyyymmdd]	Time [UTC]	Satellite
20110807	17:18	TS-X
20120826	17:18	TS-X
20120510	05:59	TS-X
20120418	05:59	TS-X
20140120	17:10	TS-X
20140122	05:51	TS-X
20140420	05:50	TS-X
20150512	05:49	S1-A
20150522	17:16	S1-A
20150527	17:25	S1-A
20150603	17:16	S1-A
20150605	05:49	S1-A
20151018	17:25	S1-A
20151030	17:25	S1-A
20151111	17:25	S1-A
20151123	17:25	S1-A
20151212	17:16	S1-A
20160307	05:49	S1-A
20160310	17:25	S1-A
20160322	17:16	S1-A
20160329	17:16	S1-A
20160516	17:16	S1-A
20160528	17:16	S1-A

TABLE II. List of TerraSAR-X and S1A scenes with wake features used in this study.

232 m, 40 m, 50 m, 70 m, and 100 m height, while wind speed measurements are taken at 33 m,
233 and from 40 m to 100 m height in 10 m steps. The wind direction is also measured at 33
234 m, and from 40 m to 90 height every 10 m steps. These parameters are provided every 10
235 minutes. Hourly measurements for water temperature at 3 m, 6 m, 10 m, 15 m, 20 m and
236 25 m depths are provided and Sea Surface Temperature (SST) measured by a directional
237 waverider buoy²⁸ at 0.5 m depth is delivered every 30 min.

238 FINO-1 observations were not available for all satellite acquisitions, and in order to fill
239 these gaps numerical model data described in the next section were used in addition.

240 D. Auxiliary dataset

241 The German Weather Service (DWD) provides hourly weather data based on observations
242 and numerical models. The variables available for our study were SST, air temperature at 2
243 m above the sea surface (T_a), and the wind vector at 10 m height. The data originate from

244 the icosahedral nonhydrostatic (ICON) model and were provided on a $1/16^\circ$ grid. It was
 245 found that overall the DWD parameters are in good agreement with the FINO-1 observations
 246 confirming previous studies²⁹.

247 The distribution of wind conditions at the time of the satellite acquisitions used for wake
 248 analysis in this study are shown in Fig. 2. It can be seen that these scenes, which all show
 249 wake features, cover a wide range of different wind directions and wind speeds. The satellite
 250 data also represent different seasons with a slightly stronger weighting of the summer half-
 251 year. This has to do with the higher probability of stable atmospheric conditions in this
 252 period, which favours the formation of wakes, as discussed in the following sections.

253 IV. METHODS

254 A. SAR wind speed retrieval

255 For the radar wavelengths and incidence angles of typical SAR instruments like TerraSAR-
 256 X or Sentinel-1 the Bragg wavelengths appearing in eq. 1 are of the order of a few centimeters
 257 (see table II). Because the energy on this roughness length scale is highly dependent on the
 258 near surface wind, these radar instruments allow the retrieval of wind speed information.
 259 The method that enables the conversion of the NRCS to near surface wind speed is based
 260 on a so called geophysical model function (GMF), which is an empirical model originally
 261 developed for scatterometer wind vector retrieval^{30–33}. Two different GMFs have been used
 262 for TerraSAR-X and Sentinel-1 scenes as the NRCS depends on the radar frequency. For
 263 C-band and VV polarization, the CMOD5 GMF³⁴ has been used. As it has been found from
 264 statistical studies that CMOD5 GMF underestimates retrieved wind speed by roughly 0.5
 265 m/s³⁵, the updated version CMOD5.N GMF³⁶ is employed for this study. CMOD5.N GMF
 266 provides 10 m wind speed at neutral conditions while avoiding errors related to atmospheric
 267 stratification. Similarly to CMOD5, X-MOD2 GMF has been tuned and adapted for X-band
 268 frequency by Li and Lehner³⁷. Both GMFs are valid for incidence angles between 20–45°
 269 and share the same functional form given by^{34,37}

$$270 \quad \sigma^0 = B_0(1 + B_1 \cos \phi + B_2 \cos 2\phi)^{1.6}, \quad (4)$$

271 where B_0 , B_1 and B_2 are functions of wind speed and incidence angle, and ϕ is the angle
 272 between the wind direction and the SAR look direction.

273 In practice, the wind speed retrieval requires some preprocessing of data such as the
 274 radiometric calibration to σ^0 . Both C-band and X-band scenes were calibrated to σ^0 using
 275 the SNAP software tool³⁸ provided by the European Space Agency (ESA).

276 According to eqs. 1 and 2 the radar cross section is depending on the 2D small scale
 277 slope distribution, which in turn is a function of the 2D wind vector. The inversion of eq.
 278 4 therefore requires knowledge about the wind direction. There are approaches to obtain
 279 wind direction information from SAR images^{39,40}, however with the additional presence of
 280 wakes and radar signatures from wind turbines⁴¹, the application of such methods to the
 281 scenes analysed in this study is not straightforward. Also, there is no guarantee that the
 282 image features required for the application of this method are actually present in the SAR
 283 scenes. For this reason we have made additional use of wind direction information from the
 284 operational German Weather Service (DWD) forecast model (see Section III D). It should be
 285 emphasized that this is of course a significant simplification, because possible wind direction
 286 changes on a smaller scale within the wake area are neglected. The general sensitivity of SAR
 287 wind speed measurements to wind direction errors was discussed in previous studies⁹. We
 288 do not discuss this point in more detail here, because of the lack of independent information
 289 on wind direction within the wake area. The overall good agreement of the alignment
 290 of different SAR image features like atmospheric boundary rolls with wind directions from
 291 atmospheric models was demonstrated in previous studies⁴². Different problems encountered
 292 when comparing numerical models with observations at higher resolutions were discussed in
 293 earlier publications^{43–45}.

294 Another issue is the so called speckle noise, which can deteriorate SAR wind speed esti-
 295 mates, if very small spatial scales are considered⁷. A usual way to remove the speckle noise
 296 is to block-average the NRCS to coarser resolutions³⁹, i.e., the calculated wind speed cells
 297 have a larger size than the initial NRCS cells. In our study the available SAR data were
 298 smoothed down to a resolution of about 200 m both in azimuthal and range direction. This
 299 approach is consistent with previous studies^{37,39} and represents a reasonable compromise
 300 between speckle noise reduction and preservation of spatial resolution required to analyze
 301 offshore wind farm wakes.

302 The empirical models discussed above are tuned to 10 m wind speed for practical reasons.
 303 However, as explained in Section II B, the radar cross section is actually closer related with
 304 the friction velocity u_* , which describes the stress at the water surface³³. The relationship

of u_* and U_{10} is of relevance for the discussions to follow and is therefore briefly addressed.

A first estimate of u_* can be obtained by assuming neutral conditions, where the wind speed profile is given by

$$u(z) = \frac{u_*}{\kappa} \log\left(\frac{z}{z_0}\right), \quad (5)$$

with Karman constant κ and roughness length scale z_0 . Using the Charnock equation

$$z_0 = 0.015 \frac{u_*^2}{g} \quad (6)$$

to relate z_0 to u_* , one obtains an implicit equation for the friction velocity given a wind speed at a certain height z .

By combination of a GMF with eqs. 5, 6, one obtains a relationship between friction velocity and normalised radar cross section as illustrated in Fig. 3. The curves correspond to X-band (dotted lines) and C-band (solid lines) for different incidence angles (20° , 30° and 40°) with the antenna looking in the upwind direction. The radar cross section increases with growing friction velocity and decreases with growing incidence angle.

Fig. 3 also gives an idea about the relevance of the radiometric accuracy of the SAR instruments. The absolute radiometric accuracy for both SAR systems is reported to be around $0.5\text{dB}^{46,47}$. For example, for C-band, 30° incidence angle, and $u_*=0.3 \text{ ms}^{-1}$ this would result in an absolute error of about 10% (see Fig. 3). One has to take into account however, that for the analysis performed in this study the relative accuracy, which is usually higher than the absolute accuracy, is of more significance. From Fig. 3 one can see that the sensitivity of wind estimates with respect to radiometric errors is higher at very high speeds. However, the very high wind speed regime is not of interest for this study anyway, because wind turbines are switched off in those conditions.

B. Image selection procedures

This study concentrates on a data set of SAR scenes collected both from X-band (TerraSAR-X) and C-band (Sentinel-1A) radar in VV polarisation. The main reason for this choice of polarisation is that the commonly GMFs were tuned to radar data acquired in this mode. Furthermore, the GMFs are known to be applicable only within an intermediate incidence angle range of approximately 20° to 65° . Apart from these restrictions on radar parameters, the following selection criteria were used:

- 334 • The scene contains the offshore wind park Alpha Ventus.
- 335 • The scene shows a wake feature behind the wind park.
- 336 • The major part of the wake feature is contained on the image.
- 337 • The scene is not affected by strong modulations of the radar cross section, which are
- 338 not related to wakes, e.g., associated with convective cells or atmospheric fronts.

339 Wakes downstream offshore wind farms lead to characteristic modulations of the normalised
 340 radar cross section visible on SAR imagery; for instance, shadow areas in NRCS indicate
 341 reduced wind speed (see Fig. 4). Unlike the ocean circulation effects for the Horns Rev wind
 342 park discussed in Hasager *et al.*¹¹, the bathymetry of the Alpha Ventus site is relatively
 343 uniform and also farther offshore, i.e., ocean current related phenomena are expected to be
 344 very weak. Also, NRCS modulations associated with ocean currents should show a strong
 345 alignment with the tidal currents. This could not be observed for the analysed data set,
 346 where the alignment was clearly dominated by the wind direction. Out of 180 scenes, which
 347 contain the wind park Alpha Ventus, 61 images showed wake features. In 42 cases the
 348 complete wake was on the image allowing a wake length estimation. From this data set
 349 9 images had to be dropped, because of strong background variations of the wind field
 350 (e.g., convective cells, boundary layer rolls), which did not allow a reasonable estimation
 351 of reference wind speed profiles. Another 10 cases had to be discarded, because insitu
 352 information on atmospheric stability was not available. In the end, 23 wake scenes were
 353 available for the analysis. As will be shown in following, this relatively small data set was
 354 sufficient to establish a relationship between atmospheric stability and wake length.

355 C. Stability parameter estimation technique

356 In this section the techniques to quantify the stability of the atmospheric boundary
 357 layer are described. Basically, the considered parameters quantify the relative role of two
 358 mechanisms: thermal stratification, which dampens turbulence; velocity shear, which creates
 359 turbulence. One of the standard parameters used in this context is the Monin-Obhukhov
 360 length scale L , which is defined as

$$361 L = -\frac{\theta_v u_*^3}{\kappa g w' \theta'_v}, \quad (7)$$

362 where θ_v is the potential virtual temperature and $\overline{w'\theta'_v}$ is the vertical flux of potential virtual
 363 temperature. When $L < 0$, the surface layer is statically unstable, and when $L > 0$
 364 the surface layer is statically stable. The absolute magnitude of L indicates the deviation
 365 from statically neutral state, with smaller $|L|$ values corresponding to larger deviations
 366 from neutral conditions. When L is negative and $|L|$ is small, buoyant processes dominate
 367 the production of turbulent kinetic energy compared with shear production. For neutral
 368 conditions one has $L \rightarrow \infty$. Based on the Monin-Obhukhov length the non-dimensional
 369 stability parameter η defined by $\eta = z/L$ with a given reference height z is often used in the
 370 literature.

371 In practice the momentum and heat fluxes in the definition of L are hard to estimate. For
 372 this reasons there are approximations for η based on the bulk Richardson number, which
 373 is easier to obtain from observations commonly available. The bulk Richardson number as
 374 used in Christiansen and Hasager³ and Hansen et al.⁴⁸ is given by

$$375 \quad R_{ib} = \frac{g}{T_2} \frac{\frac{T_2 - T_1}{z_2 - z_1} + 0.01 \frac{\text{K}}{\text{m}}}{(U_z/z)^2}, \quad (8)$$

376 where g is the acceleration of gravity, $T_2 - T_1$ the absolute temperature difference at two levels
 377 $z_2 > z_1$, and U_z is the wind speed at z . If the vertical gradient of absolute temperature is
 378 equal to the dry adiabatic lapse rate (≈ -0.01 K/m), the atmosphere is regarded as neutral
 379 and $R_{ib} = 0$.

380 The stability parameter can be estimated from this according to⁴⁹

$$381 \quad \eta = 10 R_{ib} \quad \text{for unstable conditions, i.e., } L_* \leq 0 \quad (9)$$

$$382 \quad \eta = \frac{10R_{ib}}{1 - 5R_{ib}} \quad \text{for stable conditions, i.e., } L_* > 0 \quad (10)$$

383 As indicated by eq. 8, the Bulk Richardson number R_{ib} can be computed from a difference
 384 of observed temperature at two levels and a single observed wind speed. In this study
 385 eq. 8 was used with $z_1 = 0$, i.e., T_1 is the sea surface temperature (SST). Based on this
 386 approach the stability parameter η has been computed using hourly data sets from the FINO-
 387 1 platform. The hourly measurements of wind speed, air temperature, water temperature,
 388 and sea surface temperature represent 10-minutes averages. For the absolute temperature
 389 T_2 and the wind speed U_z in eq. 8 the FINO-1 measurements at 50 m have been used.

390 In case there is no meteorological data sets from FINO-1 available, another possibly rough
 391 estimation of stability conditions can be obtained from weather maps⁵⁰ and SST from DWD.

392 The use of SST from DWD was only necessary for the scene on 30 October 2015. The air
 393 temperature from the weather maps was applied for the scene on 22 May 2015 and was kept
 394 separate in the statistical analysis for consistency reasons.

395 V. STABILITY DEPENDENCE OF WAKE LENGTH

396 A. Wake length by velocity deficit estimation

397 Wind speeds over the coastal ocean can show strong variations for many different reasons
 398 like, e.g., atmospheric fronts. Another typical phenomenon is the increase of wind speed with
 399 growing distance from the shore (Fig. 4a). The basic reason for this effect is the smaller
 400 surface roughness of the sea compared to land. In the coastal transition area between these
 401 two roughness regimes an internal boundary layer is formed, in which different adjustment
 402 processes concerning momentum, heat and moisture fluxes take place⁵¹. In many cases
 403 this effect leads to a significant NRCS increase from land to sea visible on SAR images³. A
 404 typical example is shown in Fig. 4, which displays a strong South-North wind speed gradient
 405 characterized by mean wind speed less than 6 m/s in the area South of AV wind park, while
 406 wind speed greater than 8 m/s are found in the northern part.

407 The velocity deficit V_d defined as

$$408 \quad V_d(x) = \frac{U_{freestream}(x) - U_{wake}(x)}{U_{freestream}(x)} \quad (11)$$

409 with downstream distance x from the wind farm is an important parameter for the wake
 410 analysis. The $U_{freestream}$ component is defined as the unperturbed wind upstream the wind
 411 turbines, while U_{wake} is the wind speed in the wake downstream the wind turbines. The use
 412 of a single value for $U_{freestream}$ representing the upstream conditions may introduce signif-
 413 icant errors in the computation of V_d due to non-homogeneous spatial variations of NRCS
 414 associated with the background wind field. To overcome this problem, another reference
 415 has been considered. The $U_{wake}(x)$ estimate for a certain distance x from the wind farm is
 416 obtained by averaging across the wake. The background wind field $U_{freestream}(x)$ required
 417 to estimate the wind speed deficit is obtained from an area parallel to the wake. The appli-
 418 cation of this method to the Sentinel-1 scene acquired on 30 October 2015 is illustrated in
 419 Fig. 4b. The row of boxes on the right represents the background variations of the wind field,
 420 whereas the row on the left is located downstream of the offshore windpark and covers the

421 wake. This method is similar to that used in Christiansen and Hasager^{3,5}, where $U_{freestream}$
 422 becomes dependent on the downstream distance as well. The length of each small box is
 423 2 km and the spatial mean of wind speed for each box is used for the computation of V_d .
 424 The wake length is here defined as the distance from the wind farm, where the deficit V_d
 425 changes from positive to negative sign. It is important to note, that it is not sufficient to
 426 look for general zero crossings of V_d , because quite often there is a zero crossing of the deficit
 427 going from negative to positive values at a distance of about 10 km, which is associated with
 428 bright radar signatures discussed in more detail in Section VI. It should also be noted that
 429 the fact that the analysis is based on a relative quantity (velocity deficit) makes the results
 430 quite robust with respect to errors in absolute wind speed estimates, e.g., due to errors in
 431 the GMFs or radiometric calibration.

432 B. Downstream wake length and velocity deficit and stability dependence

433 The 4 SAR images in Fig. 5a,b and Fig. 6a,b evidently show wake patterns. They were
 434 acquired under thermally stable and neutral conditions (see table III). The atmospheric
 435 stability condition is determined using eq. 8 and in-situ data. For the image acquired on 22
 436 May 2015, the thermal stability was derived from the difference between the air temperature
 437 from the Berliner Wetterkarte⁵⁰ and the SST from FINO-1 due to data gaps in FINO-1 air
 438 temperature data. The analysis of the wind turbine wake downstream is focused on the
 439 wake starting at the Alpha Ventus offshore wind park (AV). Figs. 5a,b represent situations,
 440 where the wakes are caused by AV alone, while in Figs. 6a,b the wake downstream AV is
 441 influenced by other nearby OWPs, such as Borkum and Trianel.

442 The latter situation with the superposition of wakes originating from different wind parks
 443 is naturally more complex. In the following we therefore take a two step approach, where
 444 isolated wakes are considered first.

445 1. *Isolated wakes*

446 The cases where an isolated wake without interference with other OWPs is found behind
 447 AV are referred to as “AVa” in the following. Figs. 5a,b show a TS-X scene acquired on
 448 10 May 2015, 05:59 UTC and a Sentinel-1 scene acquired on 30 October, 17:25 UTC. For

449 the TS-X image (Fig. 5a) the wind was from the southerly directions and this time the
 450 wake signature is characterized by increased σ^0 values in the downstream direction. This
 451 unexpected behavior will be addressed in more detail in Section VI. For the Sentinel-1 image
 452 (Fig. 5b) the wind was coming from south easterly direction and 2 zones with low amplitude
 453 of σ^0 (about -20 to -18 dB) can be seen downstream of the wind parks AV and the group
 454 of wind parks comprised of Borkum Riffgrund and Trianel. In the first case the wind is
 455 approximately parallel to the wind turbine rows and 3 distinct wake patterns are observed
 456 downstream, before they are merging about 3.6 km from the wind farm, which represents
 457 $\sim 30D$, where $D \approx 120$ m is the rotor diameter. This observation is consistent with the study
 458 presented in Frandsen et al.¹⁵, where this is explained by lateral dispersion of the individual
 459 wakes. In Fig. 5b the situation is different, because the wind is oblique to the rows of the
 460 wind turbines and the cross section of the resulting superposition has a more homogeneous
 461 cross section.

462 The corresponding near-surface U_{10} wind fields obtained with the method described in
 463 Section IV A are shown in Fig. 5 c,d. For the Sentinel-1 image taken on 30 October 2015 the
 464 estimated wind speed within the wake is about 7 m/s. The wind speed in the background
 465 wind field is around 2 m/s higher. For the TS-X image acquired on 10 May 2012 the
 466 estimated wind speed within the wake is around 9 m/s and thus about 2 m/s higher than
 467 in the background wind field. Again, this unexpected behaviour is analysed in more detail
 468 in Section VI.

Using the technique described in Section V A, the wake lengths were estimated as 32 km
 for the TS-X scene (Fig. 5a) and 28 km for the Sentinel-1 scene (Fig. 5b). The corresponding
 velocity deficit curves as a function of the distance from the AV wind park are plotted in
 grey in Fig. 7a. Considering the atmospheric conditions for both scenes, the longest wake is
 found for the more stable case on 10 May 2012 (Fig. 5a) with air/sea temperature difference
 of more than 5°C (table III). The corresponding maximum velocity deficit is approximately
 5% (Fig. 7a). For the Sentinel-1 scene the conditions were close to neutral with air/sea
 temperature difference of about 1°C. The observed vertical profile of temperature (and wind
 speed) from the FINO-1 platform for the case from 30 October 2015 is shown in Fig. 5f and
 indicates the instability of the atmosphere. In this case the velocity deficit is about 6%. It
 should be mentioned that a considerable number of these FINO-1 wind profiles have a quite
 complicated structure, which only in a coarse approximation follow a simple log law. This

issue has been the subject of previous studies⁵². It is interesting to note that the bright features characterised by negative deficit values in Fig. 7a occur in the more stable situation with higher air/sea temperature difference. This issue will also be addressed in Section VI. The analysis was extended to 8 SAR scenes showing isolated wakes behind AV. The data cover a wide range of thermal stability conditions, i.e, from stable to unstable. A scatter plot showing the temperature gradient and the wake length on the horizontal and vertical axis can be found in Figs. 5e. The linear regression line is given by

$$\lambda_{wakes} = 19.5 \text{ km} + 95200 \frac{\text{m}^2}{\text{K}} \frac{\partial \theta}{\partial z} \quad (12)$$

469 with wake length λ_{wakes} . It can be seen, that longer wakes are found in neutral and stable
470 conditions, whereas shorter wakes are associated with unstable conditions.

471 2. *Superimposed wakes*

472 The cases where a superposition of wakes from different OWP is observed are referred
473 to as “AVa+” in the following. The potential influence of the Borkum and Trianel wind
474 farms on Alpha Ventus is illustrated in Fig. 6. In the Sentinel-1 scene taken on 22 May 2015
475 (Fig. 6a) the wind is from the south west and AV is within the wake of the Borkum OWP,
476 which is less than 5 km to the west. The atmospheric stability during the acquisition time
477 was quite high with an air/sea temperature difference of almost 4°C (see table III). The
478 corresponding SAR derived wind field is shown in Fig. 6c. The wake length estimated for
479 this case is about 68 km and the corresponding maximum velocity deficit is approximately
480 20%.

481 On the Sentinel-1 image acquired on 28 May 2016 (Fig. 6b), the wind is in the oppo-
482 site direction compared to the previous case, i.e., from the north east. The image shows
483 bright features downstream AV before crossing Borkum wind farms. The scene was taken
484 under stable conditions with air/sea temperature difference of almost 1.1°C. The stability
485 conditions can also be appreciated from the vertical profile of the temperature (Fig. 6f).
486 The wake length estimated for this case is about 70 km and the maximum velocity deficit
487 is about 16%.

488 The corresponding velocity deficit curves as a function of the distance from the AV wind
489 park are plotted in black in Fig. 7a. It is evident that the two cases with superimposed

490 wakes lead to more pronounced deficits and longer wakes. Furthermore, it can be seen as in
 491 the previous examples (AVa) that the bright features immediately downstream AV are more
 492 pronounced in the stable situation on 22 May 2015.

493 A total of 15 SAR scenes were used for the analysis of “AVa+” scenarios with superpo-
 494 sition of wakes. The longer wakes are identified in neutral and stable conditions, whereas
 495 shorter wakes are found in unstable conditions (Fig. 6e). This case is more complex than
 496 previously, because some scenes in unstable conditions display longer wake length than those
 497 in neutral or thermally stable conditions. The difference of the wake length for the same
 498 class of stability could be due to some parameters such as the number of wind turbines as
 499 well as the layout of the wind farm, the wind speed and the wind direction. The wake length
 500 is longer than in “AVa”, which could suggest that the number of wind turbines influence the
 501 extension of the wake length. In a consistent manner as in the “AVa” cases, the dependence
 502 of V_d on the stability is not clearly established (Fig. 7a). This is also not surprising because,
 503 at least at hub height, the maximum deficit is expected to occur immediately downstream
 504 the wind park and should mainly depend on the thrust coefficient of the turbines and the
 505 wind speed⁵³. Nonetheless, for the unstable cases, the maximum of V_d is identified relatively
 506 in the first 10 km, while shifted further away from the hub for neutral and stable situations.
 507 Higher velocity deficit values are found for these complex cases.

508 Another issue to be considered is the shadowing of the FINO-1 platform by AV^{54,55}.
 509 For easterly wind directions the presence of AV leads to increased turbulence levels and
 510 reduced wind speeds compared to unperturbed conditions. Reduced wind speeds increase
 511 the magnitude of the Richardson number and hence the stability parameter $\eta = \zeta/L$ (eq. 8).
 512 The effect of AV on the temperature gradients measured at FINO-1 is more complicated. It
 513 seems to be reasonable to assume that the turbulence introduced at hub height and generated
 514 by the rotors leads to downward vertical mixing and thus affects the air temperature at 50
 515 m (height at which measurement is taken for our analysis) and below. This means that the
 516 magnitude of the temperature difference between 50 m and sea surface will be biased. In fact,
 517 for stable conditions, where the upper warm air flows over the cold air, the vertical mixing
 518 will bring the warm air down and the cold air up. This will lead to an increase of temperature
 519 in the lower layer and therefore reduced temperature gradient. For unstable conditions, the
 520 effects will be the other way around. The unstable cases with south-/north- easterly wind
 521 direction shown in Fig. 7b could in fact fall into this category. It is obvious that a correction

522 of these cases towards more neutral conditions would lead to a better consistency with the
 523 other cases shown in the plot. This also applies to the two longest wakes in Fig. 7b in neutral
 524 and slightly stable conditions, which should be moved towards more stable conditions for a
 525 more suitable correlation between the stability and wake length. For instance the correction
 526 for the case on 28 May 2016 could increase the thermal stratification difference and hence
 527 the vertical gradient on the profile of the temperature (Fig. 6f). The case on 30 October
 528 2015 in Fig. 5a (represented by the green dot) could be moved also to the right side of the
 529 plot and its temperature profile improved.

Dates (yyyymmdd)	Satellites	Air temperature T_a ($^{\circ}C$)	SST ($^{\circ}C$)	Wind speed at FINO (m/s)	Wind direction	Atmospheric stability	Wake length (km)	Velocity deficit (%)
20120510	TS-X	13.9	8.7	14.7	162.9	Stable	32	5
20150522	S1A	15	11.2	9	239.5	Stable	68	20
20151030	S1A	12	13	8.7	137.5	Slightly unstable	28	6
20160528	S1A	13.6	12.5	7.4	47.2	Stable	70	16

TABLE III. Wake and atmospheric parameters for four SAR data cases shown in Fig. 5a,b, and Fig. 6a,b.

530 VI. TURBULENCE RELATED SAR SIGNATURES

531 In this section a model is proposed to explain the radar cross section variations seen on
 532 SAR images of wakes behind offshore wind turbines. The most interesting feature addressed
 533 in this context is the increase of normalized radar cross section within a distance of typically
 534 10 km behind the turbines (e.g., Fig. 5a and Fig. 6a), which is found in about one quarter
 535 of the SAR scenes with wake features. This feature usually occurs in combination with
 536 a darker wake signature observed downstream of the bright features until the background
 537 cross section level is regained.

538 These observations seem to be quite paradox at first sight, because a naive application
 539 of standard SAR wind speed retrieval algorithms to imagery within a couple of kilometers
 540 downstream of offshore wind parks could lead to the unphysical conclusion that there is
 541 a general increase of wind speed up to hub height in this area. The proposed model is
 542 supposed to be a help for users of SAR data in the offshore wind context to avoid this
 543 misinterpretation.

According to the standard theory the radar cross section should be dominated by Bragg scattering at least for the typical incidence angles considered in this study. As explained in Section IIB the cross section is therefore for the most part controlled by the friction velocity u_* . The friction velocity in turn is highly dependent on the wind speed at higher levels, the vertical mixing length scales associated with turbulence, and the ocean surface roughness. For the roughness parameter z_0 a first order formulation is given by the Charnock relation eq. 6. This is an approximation, because there is actually also a wave age dependence, which we neglected in this first approach⁵⁶. In the Prandtl layer (typically first 100 m) we have

$$u_* = \frac{du}{dz}l \quad (13)$$

with the mixing length scale $l = \kappa z$ according to the law of the wall. This results in the classical log wind profile for neutral conditions eq. 5, which is characterised by a constant momentum flux in the vertical. In non-neutral conditions it is usually assumed that the mixing length scale is modified according to $l = \kappa z / \Phi$. Here, Φ is a dimensionless stability function depending on z/L , where L is the Monin-Obukhov length scale. We then have

$$u_* = \frac{du}{dz} \frac{\kappa z}{\Phi(z/L)}. \quad (14)$$

Based on the analysis of experimental data, the following form for the stability function is most often used in the literature⁵¹:

$$\Phi(z/L) = \begin{cases} 1 + 5 z/L & \text{for } z/L \geq 0 \\ (1 - 16 z/L)^{-1/4} & \text{for } z/L < 0 \end{cases} \quad (15)$$

To find a solution for u in eq. 14 the function $(1 - \Phi(z/L))/z$ is integrated over z yielding

$$\psi(z/L) = \begin{cases} -5 z/L + \beta_1 & \text{for } z/L \geq 0 \\ 2 \ln((1+x)/2) + \ln((1+x^2)/2) & \\ -2 \tan^{-1}(x) + \frac{\pi}{2} + \beta_2 & \text{for } z/L < 0 \end{cases} \quad (16)$$

with $x = (1 - 16z/L)^{1/4}$ and constants β_1, β_2 . We then get

$$u(z) = \frac{u_*}{\kappa} \left(\ln\left(\frac{z}{z_0}\right) - \psi(z/L) \right). \quad (17)$$

The constants are chosen such that $\psi(0) = 0$, which leads to $\beta_1 = \beta_2 = 0$. Because z_0 is small, we then also have $u(z = z_0) \approx 0$ ⁵¹.

569 The idea in this study is to define a new stability function, which takes into account the
 570 turbulence mechanically generated by the wind turbines. As already pointed out before,
 571 wakes occur predominantly in stable atmospheric condition. For this reason, we use the
 572 formulation for stable conditions in eq. 15 as a basis and define

$$573 \quad \Phi^{WT}(z) = 1 + z \frac{\Phi^{hub} - 1}{H^{hub}} \text{ for } z \leq H^{hub} \quad (18)$$

574 with a mixing length correction factor Φ^{hub} at hub height, which has to fulfill

$$575 \quad 0 < \Phi^{hub} \leq 1 + 5 H^{hub}/L \quad (19)$$

576 in order to increase the mixing length with respect to neutral conditions. With this stability
 577 function the mixing length behind the wind turbine is increased by a factor of $1/\Phi^{WT}$. The
 578 respective profile then follows as

$$579 \quad u(z) = \frac{u_*}{\kappa} \left(\ln\left(\frac{z}{z_0}\right) + (z - z_0) \frac{\Phi^{hub} - 1}{H^{hub}} \right), \quad (20)$$

580 where we imposed the boundary condition $u(z_0) = 0$.

581 The parameter Φ^{hub} describes the impact of the turbulence generated by the turbines on
 582 the vertical momentum fluxes. This turbulence dissipates in the downstream direction and
 583 therefore the length scales will slowly go back to their original value in the background wind
 584 field. Denoting by x the downstream distance from the turbine we formulate this process as

$$585 \quad \Phi^{hub}(x) = e^{-x^2/\sigma_{turb}^2} \Phi^{hub} + \left(1 - e^{-x^2/\sigma_{turb}^2}\right) \left(1 + 5 H^{hub}/L\right) \quad (21)$$

586 with an e-folding distance σ_{turb} after which the mixing length increase has dropped to about
 587 one third.

588 At the same time the mean wind speed U is reduced by a factor $R_0 = U/U_0$ behind
 589 the wind turbine and recovers to the original value U_0 after some distance downstream.
 590 Following the derivation of Betz law, which is based on the consideration of momentum and
 591 energy conservation, this factor is related to the thrust coefficient c_T via¹⁵

$$592 \quad R_0 = \sqrt{1 - c_T}. \quad (22)$$

593 Here, the thrust coefficient is used to relate wind speed U and air density ρ to the force F
 594 experienced by a turbine with rotor disc area A according to¹⁵

$$595 \quad F = 0.5 \rho A U^2 c_T. \quad (23)$$

596 The thrust coefficient itself is a function of wind speed as well. An empirical relationship to
 597 estimate the thrust coefficient from the wind speed is given by⁵³

$$598 \quad c_T = \frac{3.5 \frac{\text{m}}{\text{s}} (2U - 3.5 \frac{\text{m}}{\text{s}})}{U^2}. \quad (24)$$

599 For the downstream evolution of the wind speed reduction factor R , we assume a simple
 600 functional shape given by

$$601 \quad R(x) = 1 + (R_0 - 1) e^{-x^2/\sigma_{df}^2} \quad (25)$$

602 with an e-folding distance σ_{df} after which the wind speed deficit is reduced to about one
 603 third.

604 In total the model thus has five parameters: the undisturbed wind speed at hub height
 605 upstream U , the Obhukov length scale L in the background wind field, the mixing length
 606 scale amplification factor Φ^{hub} , and the e-folding distances σ_{turb} and σ_{df} .

607 Fig. 8 shows two simulations with different choices for these parameters. Fig. 8a,b are
 608 the respective downstream profiles of u_* and Fig. 8c,d show vertical wind profiles at three
 609 different locations upstream and downstream. The friction velocities were computed by
 610 inserting the Charnock equation for z_0 in the respective profile equations and then solving
 611 the resulting implicit equation for u_* for a given wind speed at hub height. As one can see,
 612 the model is able to reproduce an increase in friction velocity values immediately behind the
 613 turbine for stable conditions with $L = 50$ m (Fig. 8a,c). After the turbulence is dissipated, u_*
 614 drops and finally recovers to the original value. For a more unstable situation with $L = 200$
 615 m (Fig. 8b,d), the mixing mechanism is less effective and the friction velocity shows a drop
 616 downstream the OWF. Based on this model it is now clear that the occurrence of bright
 617 features downstream the turbines is favoured by stable conditions upstream, because in this
 618 case the mixing will lead to particularly strong increases of wind speeds at lower levels
 619 (Fig. 8c,d).

620 The model was fitted to observations using a standard cost function approach. In addi-
 621 tion to the SAR observations, measurements from the nearby FINO-1 platform were used.
 622 Denoting the wind speed measurements at 50 m and 100 m height taken at FINO-1 by
 623 U_{50}^{FINO1} , U_{100}^{FINO1} the cost function is of the following form:

$$624 \quad J(U, L, \Phi^{hub}, \sigma_{turb}, \sigma_{df}) = \int_0^{x_f} W_x (u_*^{SIM}(x) - u_*^{SAR}(x))^2 dx \quad (26)$$

$$625 \quad + W_{50} (U_{50}^{FINO1} - U_{50}^{SIM})^2 + W_{100} (U_{100}^{FINO1} - U_{100}^{SIM})^2$$

626

627 The functions W_x, W_{50} and W_{100} are supposed to control the relative weighting of the obser-
 628 vations in the minimisation process. In order to give the FINO-1 observations and the SAR
 629 measurements about equal weight we used $W_x = 1$ and $W_{50} = W_{100} = 0.001$.

630 Fig. 9 shows the friction velocities estimated from two Sentinel-1A scenes (blue curves)
 631 compared to the fitted empirical model results (red curves) for 22 May (Fig. 9a) and 30
 632 October, 2015 (Fig. 9b). The approach to estimate friction velocity from SAR as described
 633 in Section IV A was applied. Because we do not have a special GMF for conditions of
 634 mechanically generated turbulence available, CMOD5.N was used in this context as well.
 635 One can see that the model is able to capture the main features of the observed downstream
 636 profile. The case from 22 May represents a stable situation (see table III), in which the
 637 model reproduces an increase in friction velocity within the first kilometers downstream.
 638 On 30 October the conditions were unstable and the friction velocity shows the expected
 639 downstream drop in friction velocity both in the observations and the model. There are
 640 different factors that can explain small scale variations of the wind speed deficit that cannot
 641 be captured by the model as seen about 30 km downstream in Fig. 9a. The one-dimensional
 642 approach used in the model has of course limitations, because in reality there are also lateral
 643 wind variations across the wake, which are very complex, because they are for example
 644 associated with the interaction of wakes behind different parts of the wind park¹⁵. Due to
 645 the applied averaging process some of these lateral variations can affect the SAR derived one-
 646 dimensional deficit curves discussed here. In addition the method to estimate the background
 647 wind field can introduce some noise in cases, where the background wind speeds have lateral
 648 (across wake) variations.

649 We are aware that alternative explanations for the observed bright features could be
 650 considered. For example, offshore turbines are an obstacle for the airflow and hence one can
 651 expect a “blockage” effect related to mass conservation, which can lead to acceleration of
 652 the air below hub height¹⁷. However, we favour the presented theory of increased downward
 653 momentum flux for two reasons:

- 654 • The large eddy simulations (LES) we found in the literature (e.g.,¹⁷) do not show
 655 blockage effects over the distance observed here (10 km).
- 656 • According to the scientific literature¹⁵ the wake generated by the turbines should hit
 657 the water after about 10 rotor diameter (e.g., 1 km). This means the air below hub

658 height will be strongly affected by the turbulence from above and a blockage effect
659 further downstream is hard to imagine.

660 Because of the relatively simple analytical expressions used to describe the wind profiles,
661 the model is of course not able to describe the detailed airflow around the turbine blades.
662 This would require further assumptions about turbulence that could be included in future
663 extensions of the model. The other simplification that should be mentioned is that the law
664 of the wall assumption with growing mixing length at higher altitudes is only applicable in
665 the Prandtl layer, which is dominated by vertical momentum fluxes. It is well known that
666 with the increasing height, modern wind turbines are often at least partly inside the Ekman
667 layer, which is more dominated by Coriolis force and less by vertical mixing. Unified models
668 to describe the transition between the different layers are the subject of ongoing research⁵⁷.

669 VII. SUMMARY AND CONCLUSIONS

670 In this study cases of isolated wakes and superimposed wakes behind the AV windpark
671 are investigated. The statistical analysis is based on 23 SAR scenes acquired by TerraSAR-X
672 and Sentinel1-A. The study of isolated wakes (“AVa”) and superimposed wakes (“AVa+”)
673 shows evidence of wake effects reaching more than 30 km and 60 km downstream respectively.
674 For the first time, a clear statistical relationship between the wake length and stability has
675 been found for the location of the OWP Alpha Ventus based on the analysis of satellite data
676 and insitu measurements. The wake length is stability-dominated and the wake intensity
677 is more dependent on wind park configuration. The more stable the atmosphere condition
678 is, the longer the wake gets. This is consistent with the earlier study of Hasager *et al.*¹¹,
679 who analysed C-band SAR images and got wake lengths longer than 20 km downstream
680 for near-neutral conditions. One important new result of the present study is a parametric
681 model defining a quantitative relationship between stability and wake length for isolated
682 wakes. This model can be useful for readers concerned with planning of offshore wind parks
683 and as a reference for future studies.

684 In stable conditions, the maximum wake length was about 32 km for isolated wakes
685 (“AVa”) and 70 km for superimposed wakes (“AVa+”). Most of the scenes from “AVa”,
686 except the one from 30 October 2015, were acquired when Alpha Ventus was the only
687 existing offshore wind park (OWP) in the area. The analysed SAR scenes from the data

688 set “AVa+” with superimposed wakes were acquired since 2015, when additional OWP
689 (e.g., Borkum and Trianel) were erected. The downwind or upwind distance between the
690 new OWPs and Alpha Ventus is not big (between 2 km and 10 km depending on direction).
691 In Fig. 7b, the wake lengths from “AVa+” are not significantly longer than those from “AVa”
692 for the same unstable range. In fact, in very unstable conditions, the wake is expected to be
693 weak and short, so the influence of upstream OWPs do not affect much wakes downstream
694 Alpha Ventus. However, in stable conditions the wake length is much longer for “AVa+”.
695 This clearly materializes the additional effects from neighbouring OWPs, that could be
696 identified as a shadowing effects. The shadowing effects of one OWP to another for a small
697 spacing can be significant and cause turbine fatigue loads in neutral and stable condition,
698 because these conditions are favourable for the generation of long wake. On the contrary,
699 the shadowing effect is a much smaller factor in very unstable conditions as the wake is very
700 short. In contrast to the wake lengths, the analysis of the velocity deficit magnitude related
701 to the wake intensity does not show a clear correlation with the stability conditions overall.
702 However, for the “AVa+” cases, the velocity deficit magnitude for unstable situations seems
703 to be higher than for stable ones.

704 The study also showed that there is a lot of scatter in the data and some longer wakes
705 were in fact also found in unstable situations. Possible explanations for some unexpected
706 observations were proposed, such as variations in the background wind field, or shadowing
707 effects impacting the FINO-1 measurements. For the latter case the influence of the wake
708 on the measurement platform in FINO-1 is more complex due to the increase of vertical
709 mixing enhanced by the turbulence caused by the turbines. Also, the estimation of stability
710 parameters is not straightforward and the fact that the SAR measurements are more corre-
711 lated with the friction velocity at the sea surface rather than the wind speed at hub height
712 leads to additional complications.

713 One unexpected behaviour of σ^0 that is found in about one quarter of the wake scenes
714 was studied in more detail. In stable conditions the radar cross section is typically increased
715 with respect to the background values within a downstream distance of about 10 km. In
716 more unstable conditions the cross section shows a rapid drop in cross section immediately
717 downstream the OWF, which is more in accordance with expectations.

718 A model was proposed which explains this behaviour by an increased downward mo-
719 mentum flux behind the turbines due to mechanical turbulence generation. The model is

720 supposed to improve the understanding of the relationship between the SAR measurement,
721 which is predominantly connected to the friction velocity and wind speeds at higher levels
722 within the closer downstream regime, which is strongly affected by mechanically generated
723 turbulence. It can thus help to avoid misinterpretations of SAR imagery following from
724 naive application of standard SAR retrieval methods.

725 The future coverage of OWPs in German Bight around FINO-1 will further limit its
726 usefulness for the estimation of freestream conditions. Systematic and continuous monitoring
727 of expanding wind park installations are required. The study also showed a shortage of
728 measurements closer to the surface. This will be of growing importance, because interaction
729 processes between the atmosphere and the ocean are a critical factor for the understanding
730 of OWPs impact on the environment.

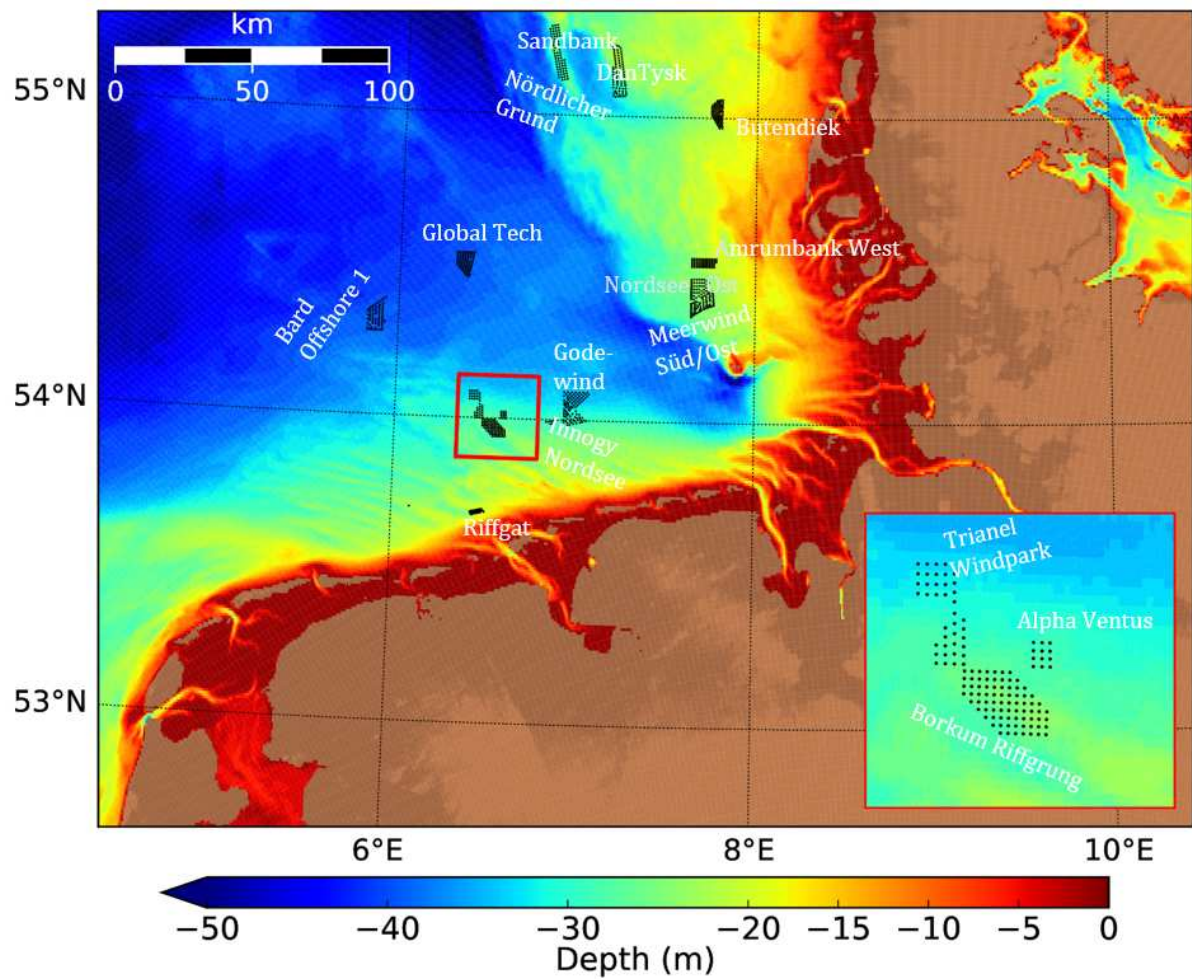


FIG. 1. Bathymetry of the German Bight with the locations of the major offshore wind parks (dated from June 2016) indicated by black dots. The red box indicates the area of interest around the offshore wind park Alpha Ventus.

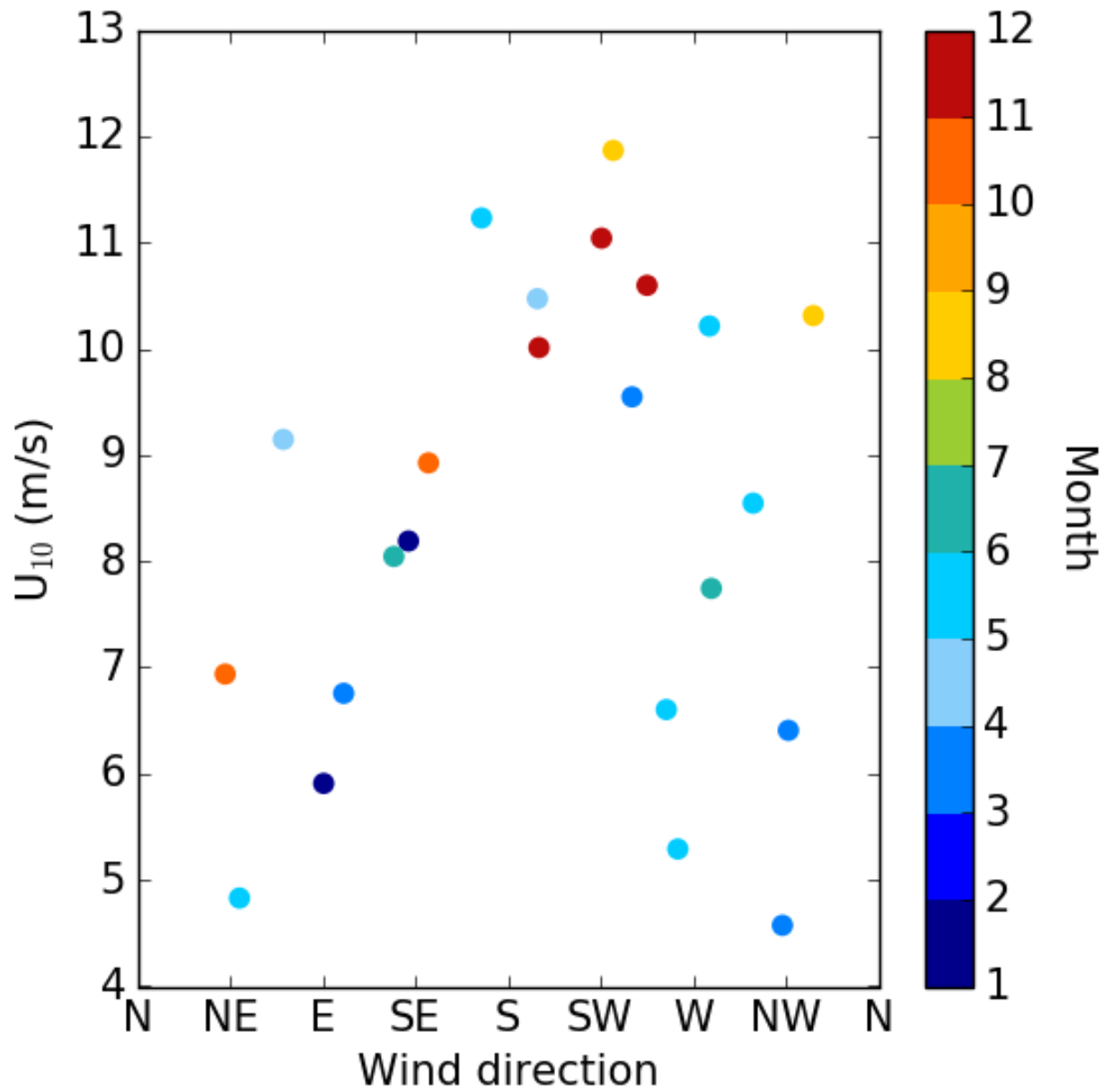


FIG. 2. Scatter plot of DWD model wind speed and wind direction for the analysed SAR scenes. The colors correspond to the month of the scene acquisition.

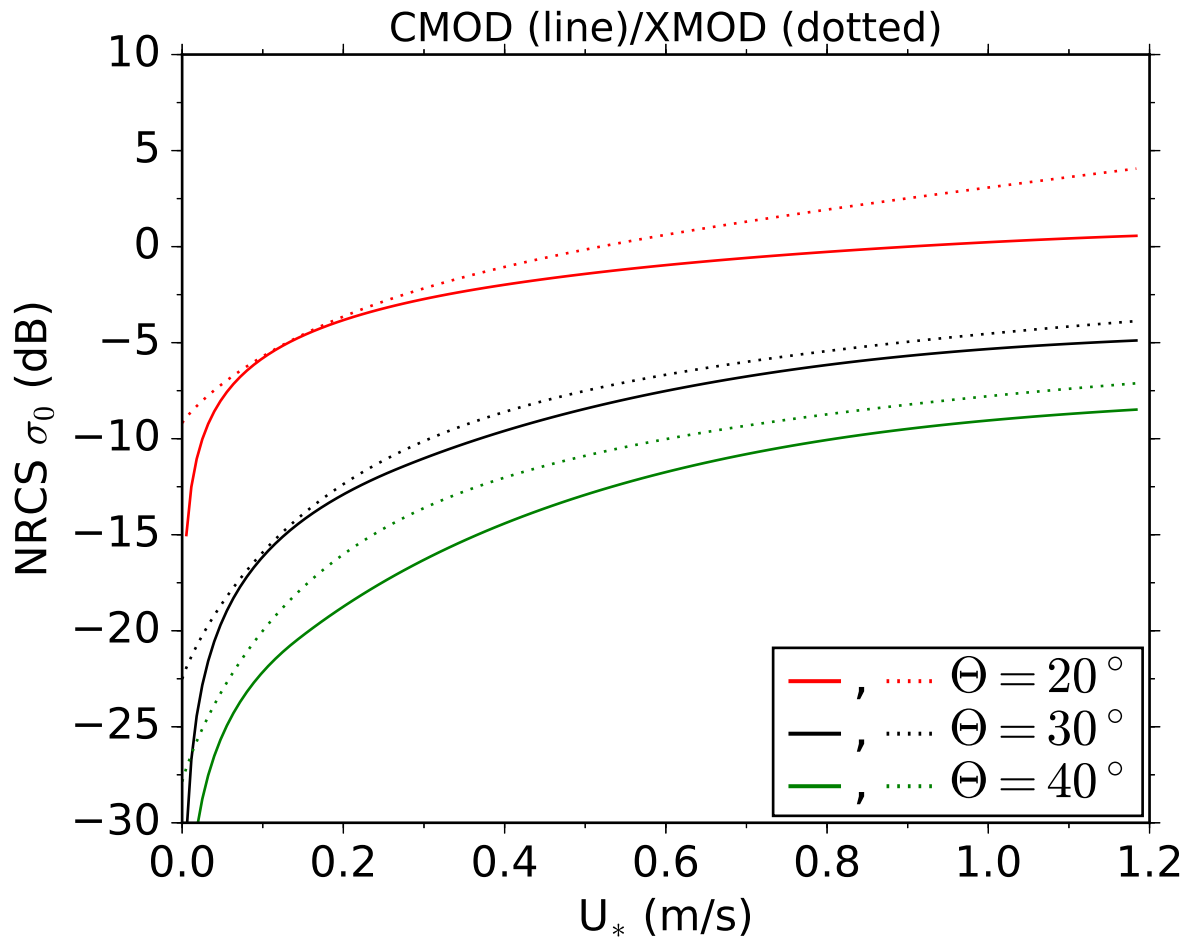


FIG. 3. NRCS as a function of the friction velocity u_* for C-band (solid line) and X-band (dotted line). The empirical CMOD and XMOD models were used for the simulation with incidence angles of 20° , 30° and 40° .

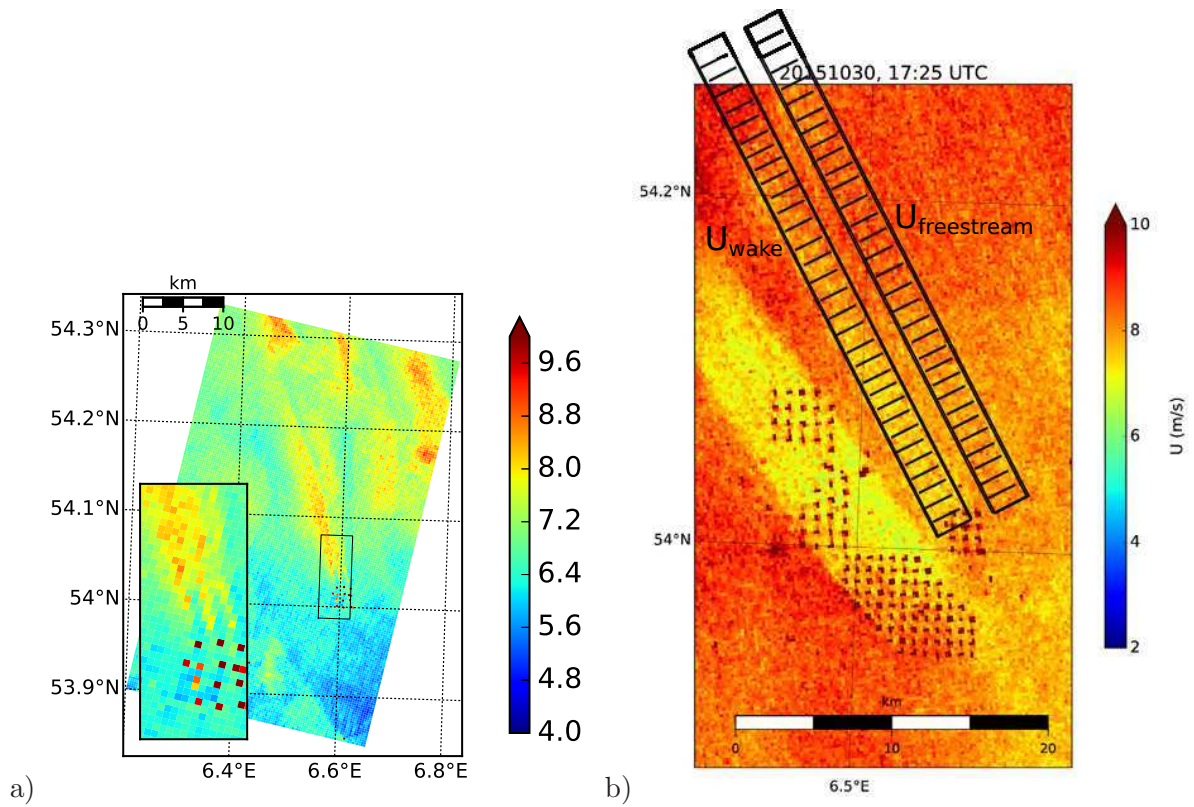


FIG. 4. (a) Example of background effects showing the increase of wind speed from South to North for an TS-X image from 10 May 2012, 05:59 UTC. b) Illustration of the method for wake length and V_d estimation based on two parallel transects applied to the Sentinel-1 image acquired on 30 October 2015, 17:25 UTC.

Impact of atmospheric stability on SAR imagery of offshore wind park wakes

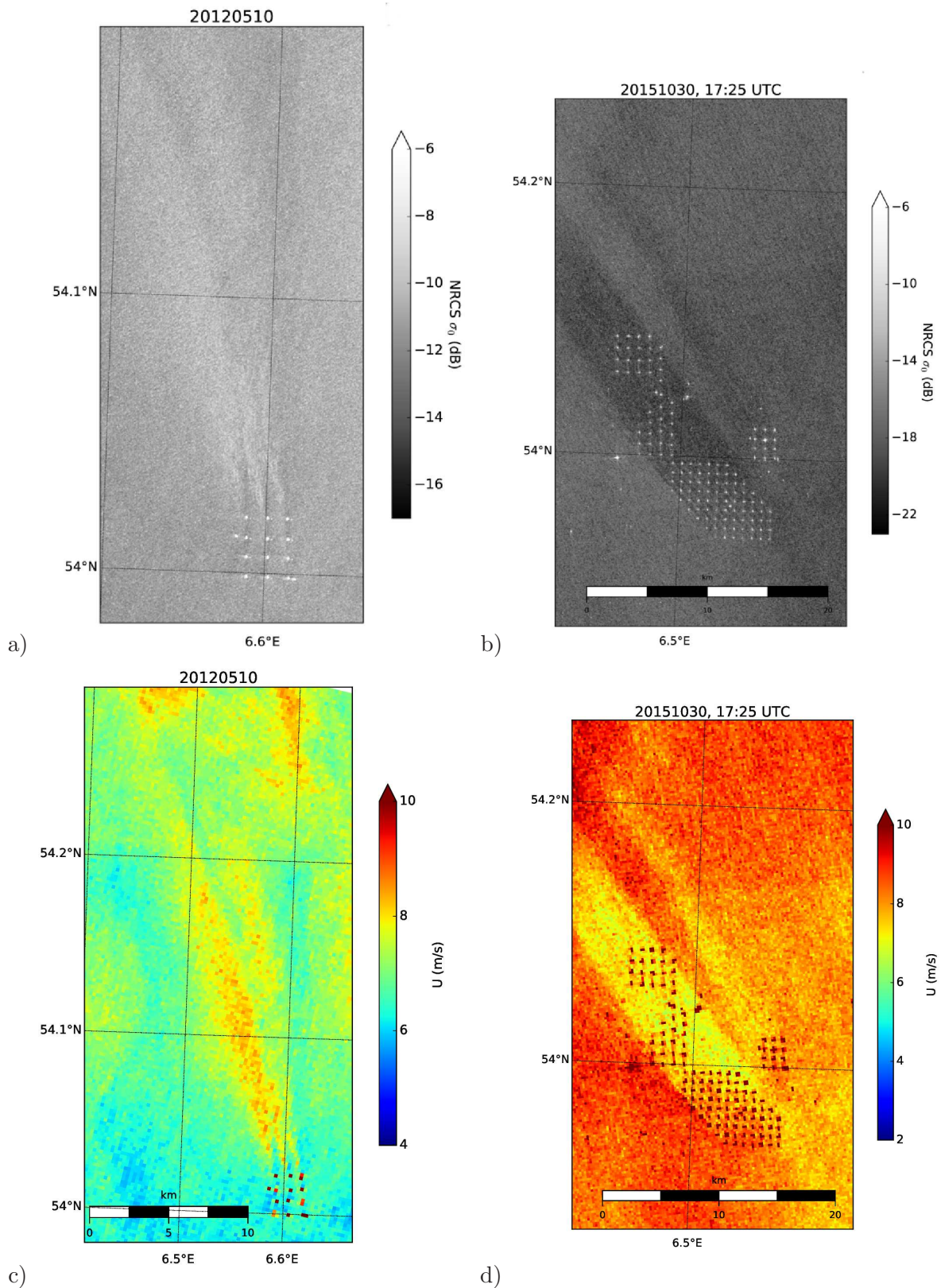


FIG. 5. (a,b): TS-X (a) and Sentinel-1 (b) scene showing an isolated wake behind AV acquired on 10 May 2012, 05:59 UTC (a) and on 30 October 2015, 17:25 UTC (b). (c,d): SAR derived U10 wind speed derived from the scenes shown in a) and b).

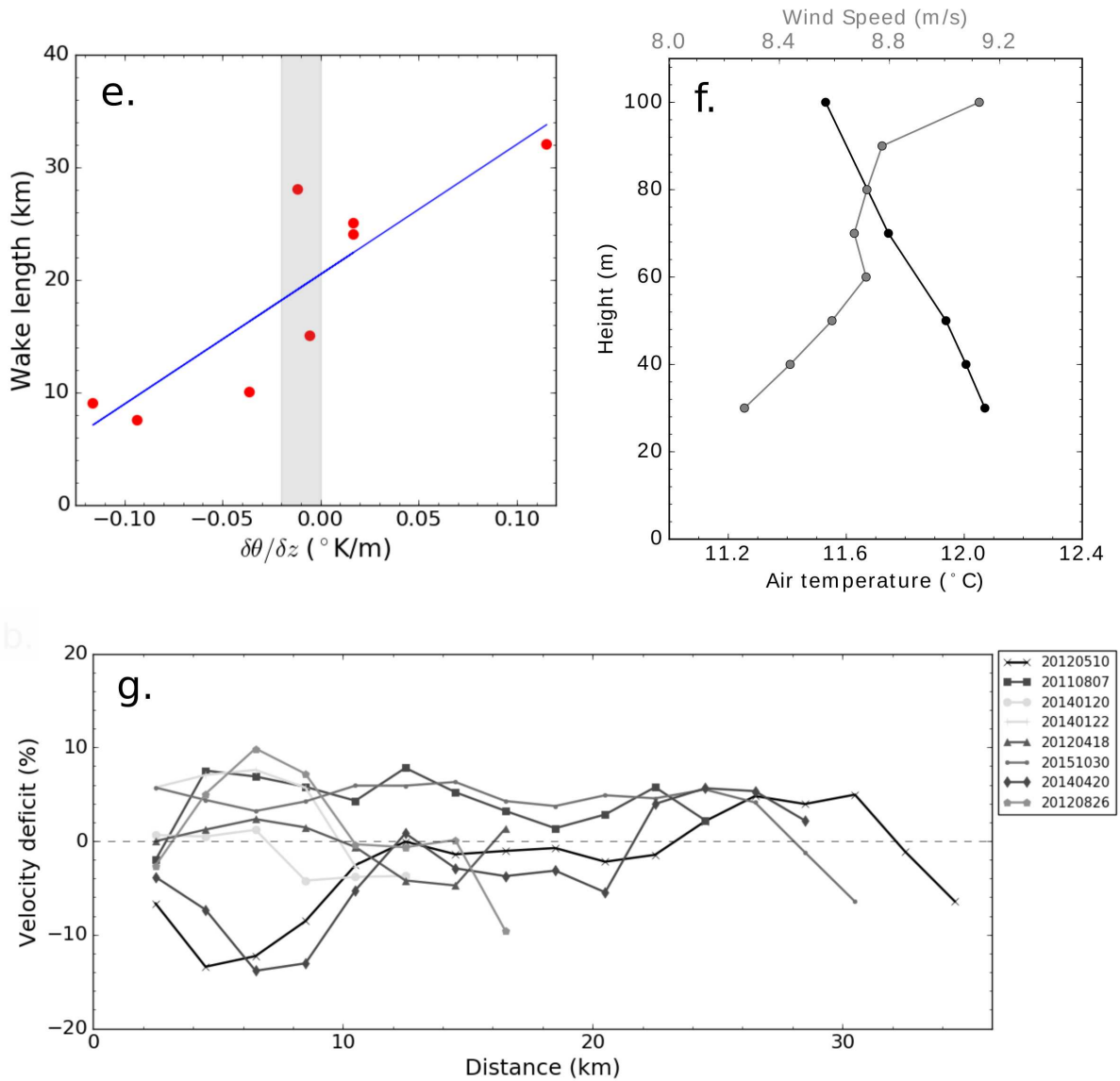


FIG. 5. (continued) (e) Scatterplot of thermal stability derived from FINO-1 insitu data versus SAR derived wake length for the “AVa” cases. (f) Vertical temperature (black) and wind speed (gray) profiles from FINO-1 for 30 October 2016. (g) Velocity deficit curves for isolated wake cases with color indicating the stability: the darker color for more stable and gray color more unstable.

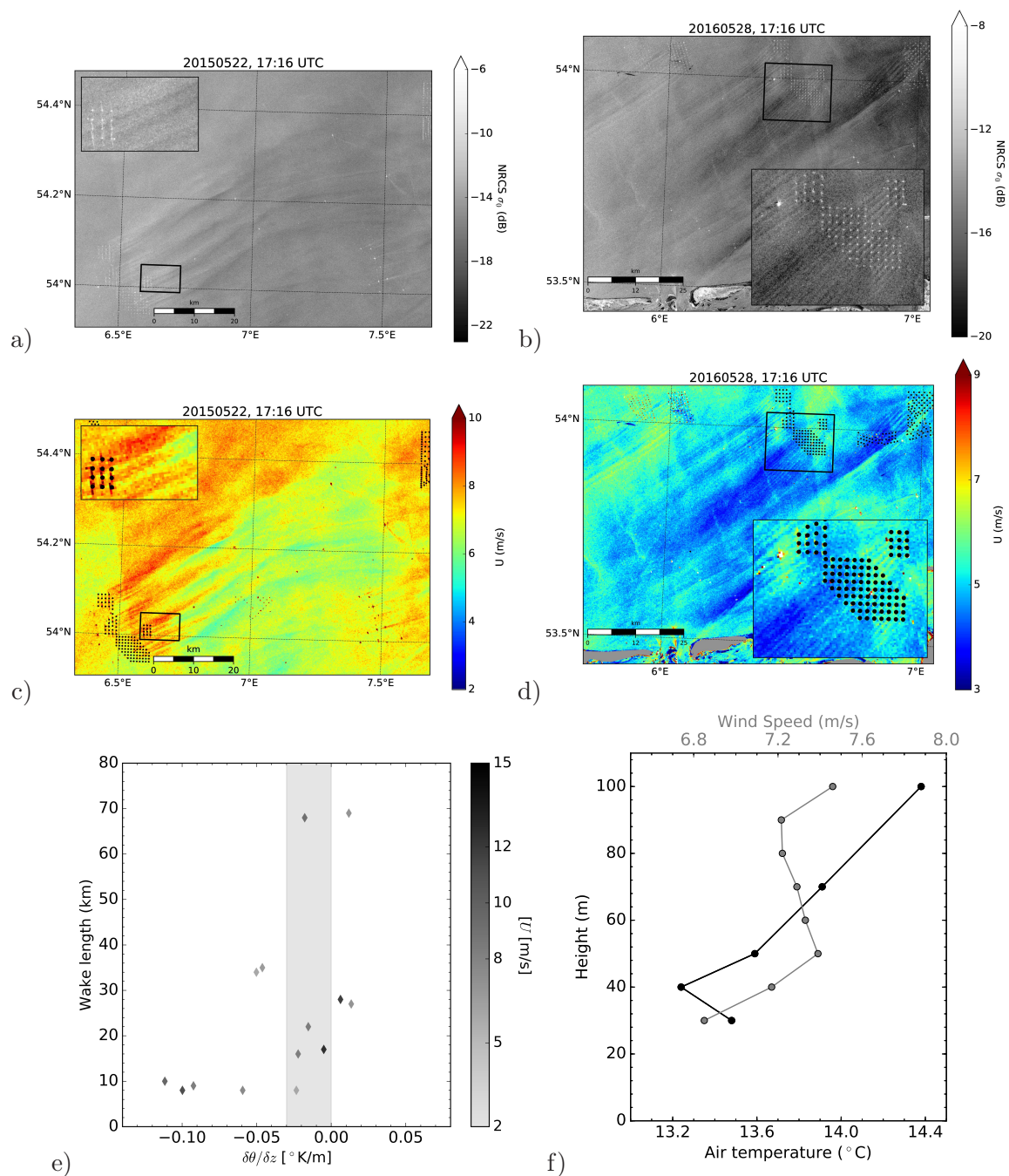


FIG. 6. (a,b): Two Sentinel-1 SAR scenes showing wake features behind the Alpha Ventus wind park acquired on 22 May 2015, 17:16 UTC (a) and 28 May 2016, 17:16 UTC (b). (c,d): SAR derived U10 wind speed derived from the scenes shown in a) and b). (e) Scatterplot of thermal stability derived from FINO-1 insitu data versus SAR derived wake length for the “AVa+” cases. (f) Vertical temperature (black) and wind speed (gray) profiles from FINO-1 for 28 May 2016.

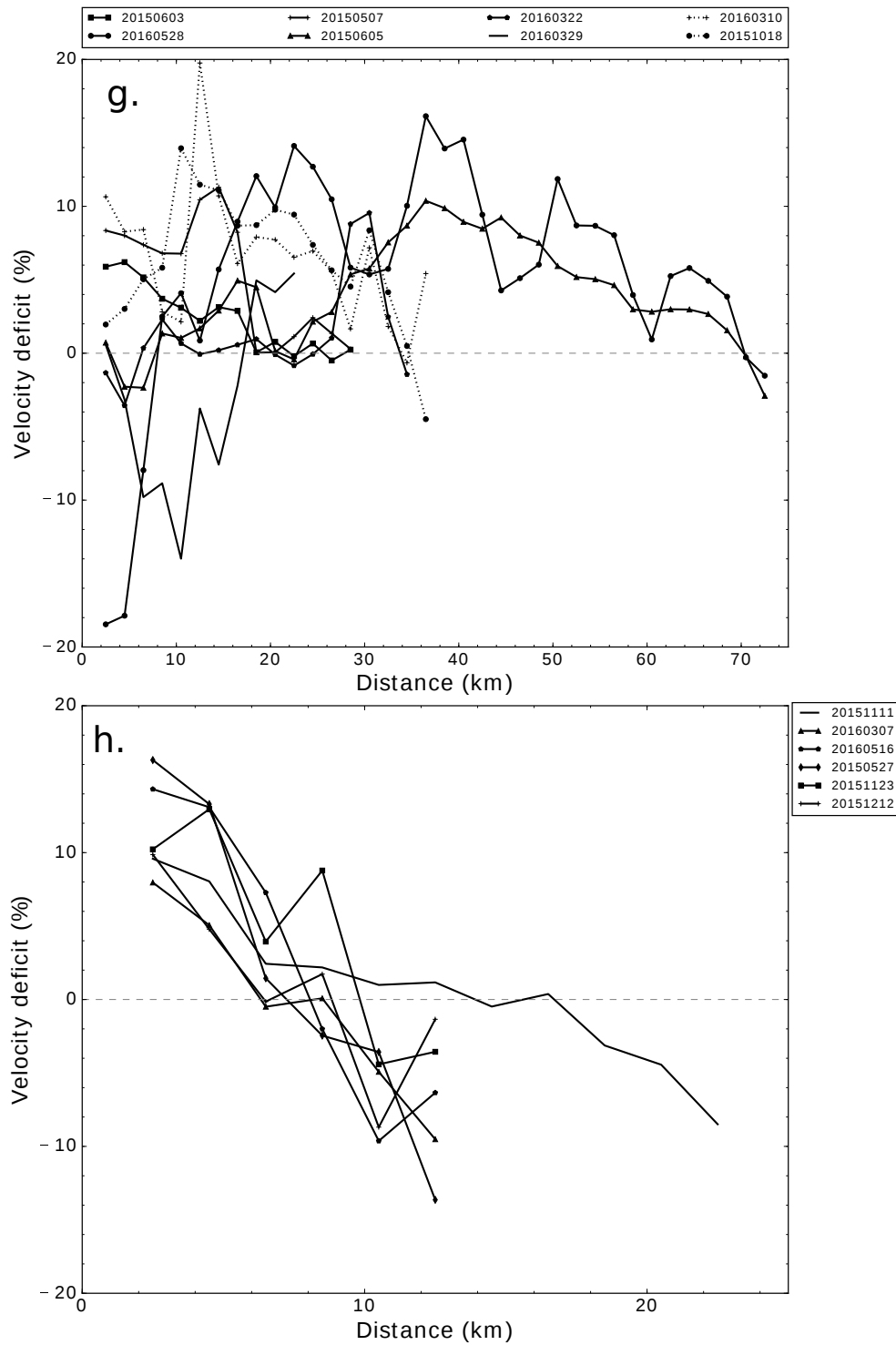


FIG. 6. (continued) Velocity deficit curves for longer (g) and shorter wakes (h) derived from SAR scenes for the “AVa+” cases. The curves in g) correspond to neutral and stable situations with the exception of the two gray curves, which are in unstable conditions.

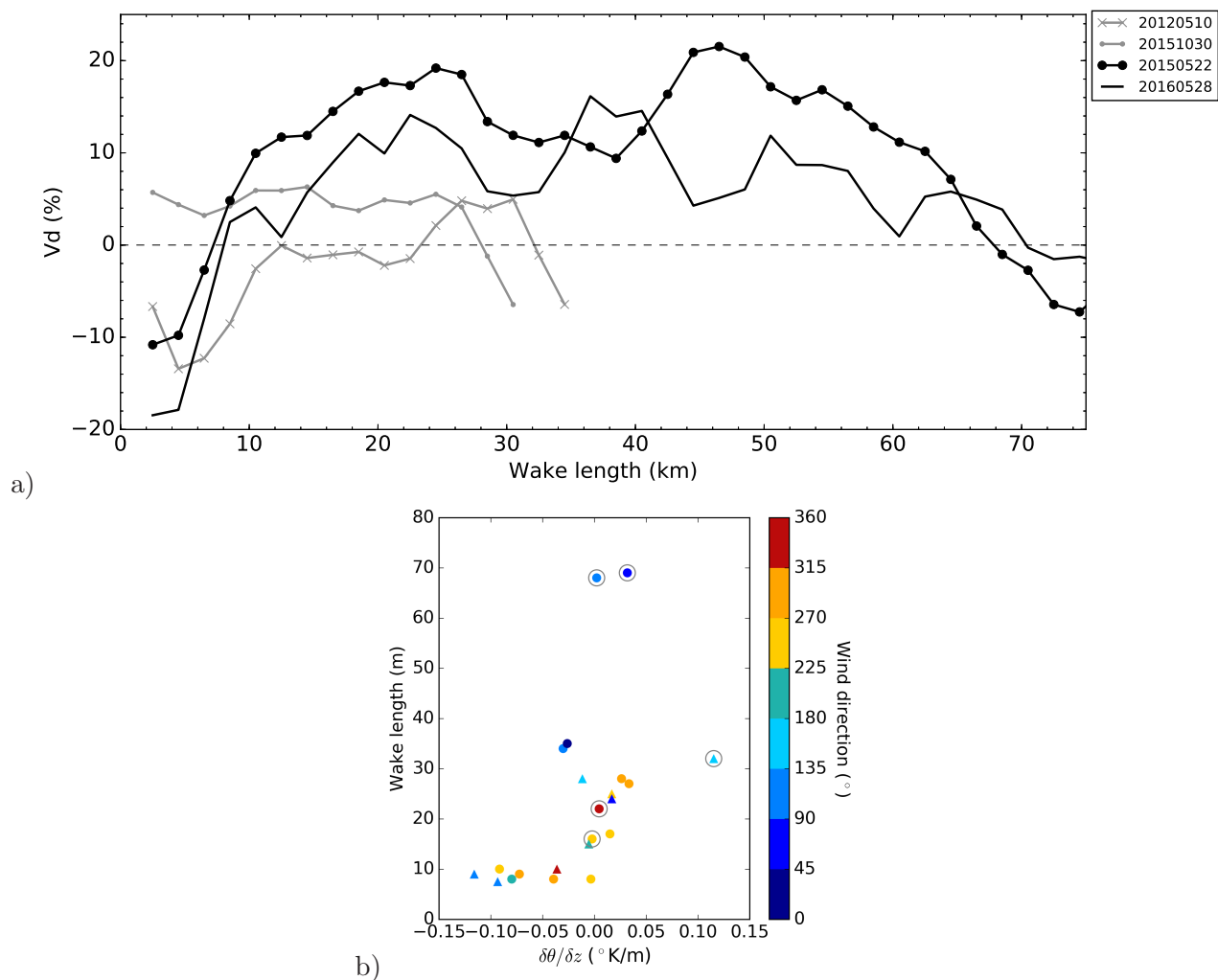


FIG. 7. (a) Velocity deficit curves derived from the SAR images shown in Fig. 5 and Fig. 6. The grey curves (10 May 2012 and 30 October 2015) refer to the “AVa” situation, while the black curves (22 May 2015 and 28 May 2016) correspond to the more complicated “AVa+” cases. (b) Scatterplot of thermal stability derived from FINO-1 insitu data versus SAR derived wake length for both “AVa+” and “AVa” cases. Triangle symbols represent the cases in “AVa” and the dot symbols are from “AVa+”. The colors represent the wind direction of each scene from DWD data. The circle represents the scenes, which show the increase of wind speed in the first kilometers downstream.

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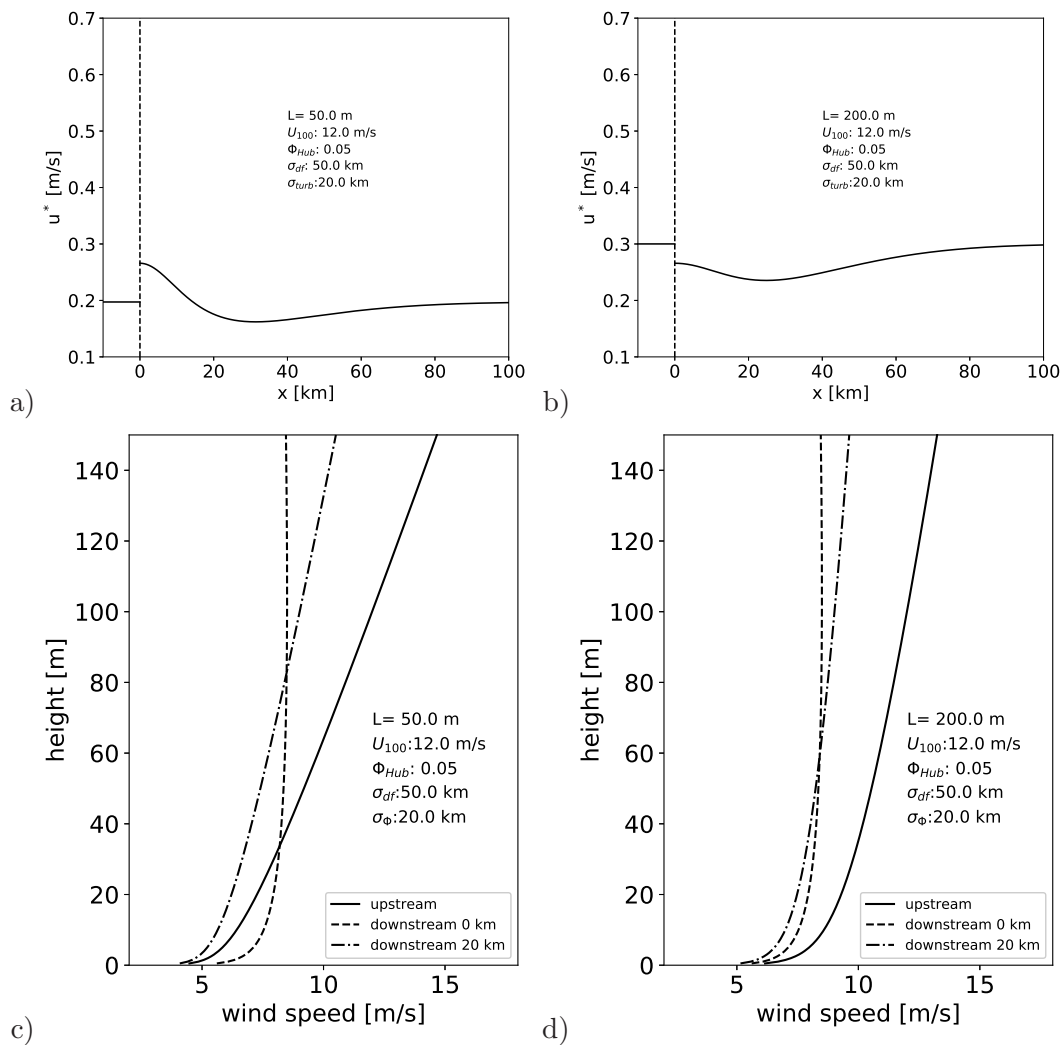


FIG. 8. (a,b) Downstream profiles of friction velocities estimated with the semi-empirical model using different parameter settings. (c,d) Corresponding vertical profiles upstream, directly behind turbine, and 20 km downstream estimated with the semi-empirical model using the parameter settings in a,b.

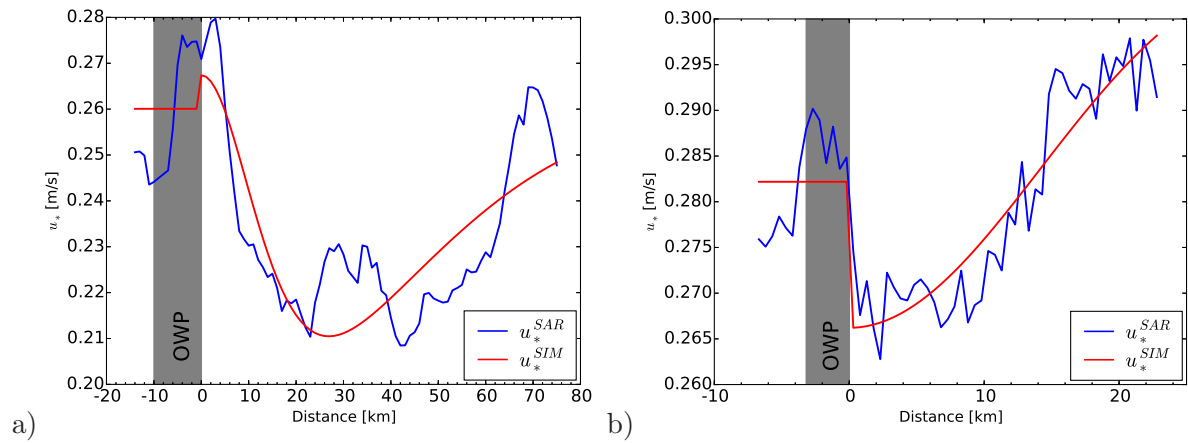


FIG. 9. Downstream profile of friction velocity obtained for the Sentinel-1a scenes acquired on 22 May (a) and 30 October (b), 2015 compared to the fitted empirical model (red curve) The gray shaded area indicates the position of the Alpha Ventus offshore wind park.

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