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Fracture prediction based on a two-surface plasticity law for the anisotropic magnesium alloys AZ31 and ZE10

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Abstract: The objective of the present study was to characterize the fracture limits of two magnesium sheet alloys, AZ31 and ZE10, using different ductile fracture criteria and to evaluate these criteria for different loading conditions in the framework of finite element (FE) simulations. A recently proposed two-yield surface plasticity model, which separates the strain contributions of dislocation glide and mechanical twinning on the (10-12) plane, was adopted to describe the strength differential effect and anisotropic hardening behaviors of the magnesium alloys. The deformation and fracture behaviors of the materials were measured in uniaxial tension, U-notched tension, and shear, thus encompassing different stress states. The fracture criteria parameters were optimized using an experiment–simulation hybrid approach. The suggested deformation and fracture models were applied to the FE analysis of thin square tubes under two loading conditions, namely, global axial tube compression and three-point bending. The simulation results were compared with those of the respective structure tests. The two-yield-surface model was found to be able to successfully reproduce the punch load–displacement responses in both cases, revealing its superior performance relative to the von Mises model. In the case of failure prediction, all fracture criteria resulted in similar predictions for tube compression. However, the failure prediction for three-point bending was found to be highly dependent on the fracture criteria, among which the MMC criterion provided the most realistic prediction. The prediction results were further analyzed by investigating the stress history and damage evolution in the critical regions of the specimens during tube compression and three-point bending.

Keywords: magnesium sheet alloy; strength differential effect; anisotropic hardening; ductile fracture criteria; finite element simulations

1. Introduction
Magnesium alloys offer great promise for application to lightweight vehicles because of their high strength-to-weight ratio. As such, they could be a practical alternative to conventional steels and aluminum alloys. Although cast magnesium alloys have already been successfully utilized for this purpose, the application of wrought alloys such as sheets and extrusions is still limited. One critical problem related to the latter is the poor ductility of the material at room temperature, which originates from its microstructural characteristics. The irreversible deformation of magnesium is, in general, controlled by a shortage of available slip systems and a resulting competition between dislocation slip and deformation twinning. It has been confirmed that mechanical (10-12) twinning is easily instigated and can make a major contribution to deformation in the case of c-axis extension, but this is not the case with c-axis compression (Christian and Mahajan, 1995; Partridge, 1967). Magnesium sheets and extrusions with a hexagonal close-packed atomic structure develop a strong (0001) basal texture during rolling and extrusion (Rollett and Wright, 2000), with the c-axis parallel to the thickness direction. The deformation along the thickness direction is therefore restricted and early failure is promoted. This trait is particularly detrimental in the case of sheet-forming operations performed at room temperature.

A reliable numerical tool for failure prediction would be beneficial to the design and optimization of the processes used to form magnesium alloys, as well as for the assessment of structures. In general, numerical failure predictions by means of finite element (FE) simulations are achieved by adopting either a failure criterion or a coupled damage model. The former method assumes that failure occurs when a scalar-valued measure of damage exceeds a critical value and that the elasto-plastic behavior of the material is not affected by this measure of damage (uncoupled models). The latter method assumes the existence of internal defects such as voids or microcracks, and defines a damage variable as the volume fraction of such defects. It is further assumed that the damage evolution influences the elasto-plastic behavior of the material and, therefore, the constitutive model includes an evolution equation for damage accumulation (coupled models).

Coupled damage models have been motivated by the failure mechanism through the nucleation, growth and coalescence of voids or microcracks e.g. (Gurson, 1977; Lemaitre, 1985; McClintock, 1968; Rice and Tracey, 1969). These processes are evidenced in case of ductile failure, with significant plastic deformation preceding the failure under tensile states. For this reason, these failure models are generally considered to be more physical than purely phenomenological. Models coupling anisotropic plasticity and damage have been developed within various levels of complexity, e.g. (Benzerga et al., 2010; Steglich et al., 2010; Stewart and Cazacu, 2011). Due to the softening effect of the progressive damage evolution, this class of models
suffers from an inherent mesh dependency of the results, which requires a regularization. Furthermore, their successful application is reported for bulk material and axisymmetric stressing, in which a considerably high stress triaxiality develops. In the deformation of thin-walled structures, in which plane stress conditions prevail and failure is dominated by shear, damage models have to be enriched by a second parameter beside the stress triaxiality characterizing the shear stress state, the Lode angle. Extensions were proposed that incorporate damage growth under low triaxiality straining for shear-dominated states (Bai and Wierzbicki, 2008; Mohr and Marcadet, 2015; Nahshon and Hutchinson, 2008). For the assessment of the formability of sheets and the crash energy absorption of thin walled structures, the use of shell elements is inevitable due to their computational efficiency and robustness. In such a framework plane stress conditions are considered, hence stress triaxiality and Lode angle depend on each other. This precludes the use of the two-parameter approaches.

Experimental investigations of the failure behavior at ambient temperatures on a magnesium alloy AZ31B were conducted by (Kondori and Benzerga, 2014a, b). By analyzing cylindrical samples cut out of a 32 mm thick plate the authors found a weak dependency of the failure strain on the triaxiality (Kondori and Benzerga, 2014b). In their follow-up investigation, the same authors reported that the ductility has its maximum at a moderate stress triaxiality. The highest strain to failure was achieved in round, moderately notched tensile bars, which render a maximum local stress triaxiality of approx. 0.76 (Kondori and Benzerga, 2014a). For lower and higher triaxialities, the tested samples revealed a reduced ductility. This effect is accompanied by a transition in the failure mode: from slanted (shear) failure to macroscopically flat fracture at higher triaxialities. It is worth noting that this “threshold value” lies above the limit of 2/3 given by the plane stress assumption. In the same vein, (Prasad et al., 2015) conducted mode I fracture tests on C(T) samples. The authors observed significant notch blunting before crack initiation and a ductile micro-void growth and coalescence mechanism in the center of the sample leading to crack extension. The sample thickness of 9.6 mm allows a considerable constraint along the thickness direction; hence, the stress state in the ligament is triaxial. A damage model based on void growth and coalescence was successfully applied for a magnesium plate of WE42 with a thickness of 32 mm. Second phase particles on and near the grain boundaries were identified in this case to act as void or crack initiation sites. With the sample geometries used, the authors were able to include stress triaxialities up to 1.4.

Investigations of damage mechanisms of magnesium sheet materials are less frequently found in the literature. Kang et al. (Kang et al., 2013) investigated AZ31 sheets of a thickness of 2 mm by means of tensile tests and texture scans. They report the development of a premature diffuse neck without any localized
necking followed by an abrupt fracture. Beside extension (10-12) twins, contraction (10-11) twins were observed in areas of strain higher than 0.10, accommodating deformation in the sheet’s thickness direction. In (Ghaffari Tari et al., 2014) the constitutive response of a 1.6 mm thick AZ31 sheet under tensile and compressive in-plane loading is explored, but the authors do not comment on the specific failure modes. The magnesium alloy ZEK100 with a sheet thickness of 1.5 mm is focused in (Abedini et al., 2017), where failure in shear loading was analyzed. The mini-shear samples the authors examined exhibited abrupt fracture. The crack initiation site could not be identified. Consequently, the equivalent strains were reported in the center of the mini-shear sample where the strains where highest. Jia and Bai (Jia and Bai, 2016) performed a comprehensive testing program on AZ31 sheets of 2 mm thickness. In line with the results of Kang et al. (Kang et al., 2013) they report a shear dominated fracture mechanism in uniaxial and multiaxial tensile loading conditions. Furthermore, they commented on a brittle nature of fracture as no apparent localized necking was observed.

In summary, the fracture behavior of thin magnesium sheets does not undergo the typical ductile fracture with diffuse necking, localized necking, and failure in sequence during a tensile test. The fracture surfaces reveal flat regions with barely visible dimples, and dimpled regions in which remainders of second phase particles are absent.

Several authors proposed approaches to tackle the challenges arising with low triaxiality in thin sheets and mesh dependency of coupled damage models. Failure criteria were suggested to assess the fracture behavior of sheets. These failure criteria use critical damage parameters derived from the elasto-plastic behavior of the material by a hybrid approach involving mechanical testing and numerical analyses. They were proposed to predict the onset of ductile fracture considering various hypotheses, each motivated by experimental observations. This class of models addresses ductility and workability (Cockcroft and Latham, 1968), metal forming operations (Clift et al., 1990), and the effects of loading rate (Johnson and Cook, 1985). More recent developments in this field have addressed the range of low or negative triaxiality (Bao and Wierzbicki, 2004), the anisotropy of the failure strain (Fourmeau et al., 2013; Luo et al., 2012), the temperature evolution with mechanical loading (Roth and Mohr, 2014), compression-tension load reversals (Marcadet and Mohr, 2015) and the role of residual stresses (Zhou et al., 2012). With increasing number of fracture criteria proposed, their evaluation and the comparison to provide a guide for the selection become challenging. Useful comparisons of the predictive capabilities for metal forming operations were carried out by Han and Kim (2003), Wierzbicki et al. (2005), and Chen et al. (2010).
The above models are computationally more efficient than coupled damage models, since they do not require internal variables. When it is necessary to find only the failure initiation point of a structure, failure prediction can simply be achieved by post-processing the elasto-plastic finite element simulation that is obtained without any information on the damage. Hereby, the failure criteria can be applied either in a 3D framework or under plane stress conditions (Lou et al., 2014). The latter is common in the case of sheet forming operations, when forming limits are concerned.

Lou et al. (2017) clearly highlighted the challenge related to the measurement of the failure strain, which should be regarded as being a local strain at the position of failure initiation. Consequently, 3D models are generally considered together with the calibration experiments to capture the loading path and the local field quantities (Luo et al., 2012). The circumstances under which the information retrieved from a 3D-analysis of the local fields in a zone of (shear) localization can be transferred to a structural assessment based on the assumption of a plane stress state, using e.g. shell finite elements, has yet to be resolved.

The objective of the present study was to assess failure of thin-walled magnesium structures. Two kinds of sheet alloys were considered, namely, AZ31 and ZE10, both with a nominal thickness of 2 mm. A recently proposed plasticity model (Steglich et al., 2016) was applied to separate the strain contribution of dislocation glide from that of mechanical twinning. Failure initiation in sheets is a local event; hence, the crack initiation in flat standard laboratory samples is studied on the basis of three-dimensional finite element simulations in order to capture the extrema of the field quantities. The fracture limits of the sheets are identified from tension (standard and U-notched specimens) and shear tests, covering different stress states in terms of triaxiality and Lode angle (Section 2). The measured fracture limits are then used to determine the parameters of Mohr-Coulomb-type fracture criteria (Bai and Wierzbicki, 2010; Luo et al., 2012) as well as other established criteria (Section 3). These fracture criteria are further evaluated in the failure prediction of thin, square-tube compression (crushing) and three-point bending tests (Section 4), featuring large amounts of heterogeneous deformation. Since simulations in metal forming and structural assessment of this kind are commonly conducted assuming a plane stress state, the applicability and the predictive power of the models are critically discussed in terms of critical global loads, deformation modes and failure positions.

2. Experimental observations

2.1. Materials
Two commercial magnesium alloys were selected for investigation, both provided as thin rolled sheets. The commonly used AZ31 (Mg + 3%Al + 1%Zn) and ZE10 (Mg + 1%Zn + 0.3%mischmetal, a cerium-based alloy of rare earth elements) alloys were selected for the present study. Both sheets had a nominal thickness of 2 mm. The alloys were chosen due to their excellent availability and their potential for use in structural applications, notably load-carrying structures. AZ31 exhibits a strong basal texture with a preferential alignment of the basal planes parallel to the sheet plane, whereas ZE10 exhibits a significantly weaker texture. The differences in the mechanical behavior and the formability of these two alloys have been described in previous studies (Bohlen et al., 2007; Mekonen et al., 2012; Yi et al., 2009). ZE10 exhibits superior ductility at room temperature relative to AZ31, which is associated with the rare earth elements in ZE10 and the result of deformation and recrystallization during sheet rolling. A more detailed comparison of the microstructures of the two alloys can be found in Steglich et al. (2014). The two alloys are used in form of annealed magnesium sheets (O-temper).

2.2. Mechanical tests under different stress states

Four different experiments were conducted with both the AZ31 and ZE10 sheets: uniaxial tension of standard specimens, tension of U-notched specimens, in-plane uniaxial compression of cube specimens (prepared by stacking five square sheets), and shear of butterfly-shaped specimens. For convenience, these tests are designated UT, NB, UC and SH, respectively, in this paper. All the tests were conducted along both the rolling direction (RD) and transverse direction (TD) of the sheets. Each test was repeated two or three times to confirm the reliability of the results. The specimen dimensions and experimental procedures are described in detail elsewhere (Steglich et al., 2016; Steglich et al., 2014).

Figures 1(a) and (b) show the engineering stress-strain curves for the UT and UC tests, respectively, for AZ31 sheets. Figure 1(c) shows the force-notch opening displacement (NOD) for the NB tests, where the force was normalized by the cross-sectional area \( A_0 = 12 \text{ mm} \times \text{thickness} \) and the NOD was normalized by the initial opening distance \( l_0 = 2 \text{ mm} \). Similarly, Fig. 1(d) shows a normalized force-displacement for the SH tests with a cross-sectional area \( S_0 = 5 \text{ mm} \times \text{thickness} \). The displacement in the SH tests is based on the distance between two pins attached to a specimen to monitor the global deformation of that specimen (Steglich et al., 2016). Figure 2 shows the same experimental results for ZE10 sheets. Based on these measured stress-strain or load-displacement data, the fracture limit was defined as the onset of significant drop in the load (or stress), as indicated by the symbols in Figs. 1(a), (c) and (d) for the AZ31 sheets and Figs. 2(a), (c) and (d) for the ZE10 sheets. Both materials fail in a slanted mode in the case of the NB tests, in case of the SH samples.
with a fracture surface oriented perpendicular to the thickness direction. For the UC tests, it was impossible to identify the fracture limit because of the irregularity of the stress drops, as shown in Fig. 1(b), which resulted from buckling, partial debonding of the glued sheets. In thin sheets, failure in compression is the consequence by an instability, its avoidance requires anti-buckling devices, e.g. (Boger et al., 2005; Kuwabara et al., 2009). In the investigation used here, the stress state deviates from uniaxial compression as the buckling proceeds and the UC data cannot be used for the calibration of the fracture model in the present study.

3. Constitutive model and fracture criteria

3.1. Constitutive model

As evidenced in the previous section, the AZ31 and ZE10 sheets exhibit strong tension-compression asymmetry in yielding and hardening. The tension-compression asymmetry is due to the two deformation mechanisms, dislocation glide and twinning, being activated differently depending on the loading. Plastic deformation is a result mostly of dislocation glide during in-plane tension, whereas it is caused by both dislocation glide and twinning during in-plane compression. In the latter case, twinning is the dominant mechanism in the early stages of compression, after which dislocation glide occurs and becomes dominant. In addition, for the ZE10, significant in-plane anisotropy is observed between the RD and TD. This in-plane anisotropy is relevant to the direction-dependent distribution of the basal texture in the ZE10, as explained in Steglich et al. (2014). The present study adopts a phenomenological two-yield surface model (Steglich et al., 2016), which can capture all the tension-compression asymmetry and in-plane anisotropy, described above. This model is based on micromechanisms in that the deformation by dislocation glide and that by twinning on the (10-12)-plane (so-called tension twinning, as it is active in c-axis tension only) are handled separately using two yield surfaces. The effective stress and strain quantities are therefore defined for each yield surface, namely, \( \sigma_g \) and \( \varepsilon_g \) for the glide yield surface, and \( \sigma_t \) and \( \varepsilon_t \) for the twinning yield surface. This approach allows us to define the yielding and hardening behaviors in tension and compression, separately. A detailed formulation of the two-yield surface model is provided in Appendix A. The model parameters for the AZ31 and ZE10 are given in Table A1 and A2 (Steglich et al., 2016).

Each active deformation mechanism is described by its yield surface and hardening law. The interaction between two deformation mechanisms is realized through a coupling of the hardening laws,
\[
\bar{\sigma}_g(\bar{\varepsilon}_g, \bar{\varepsilon}_t) = R_g + H_g \bar{\varepsilon}_g + Q_{1g}[1 - \exp(-b_{1g}\bar{\varepsilon}_g)] + Q_{2g}[1 - \exp(-b_{2g}\bar{\varepsilon}_g)]
\]

\[
\bar{\sigma}_t(\bar{\varepsilon}_t) = R_t + H_t \bar{\varepsilon}_t + Q_{1t}[\exp(b_{1t}\bar{\varepsilon}_t) - 1] + Q_{2t}[1 - \exp(-b_{2t}\bar{\varepsilon}_t)]
\]

Note that the isotropic hardening of the glide yield surface is governed by both \( \bar{\varepsilon}_g \) and \( \bar{\varepsilon}_t \) to address the obstacles to dislocation motion resulting from mechanical twinning. As implied in Eq. (2), the hardening of the twinning yield surface is not affected by the glide mechanism. For a more detailed description, the reader may refer to the appendix.

### 3.2. Ductile fracture criteria

Stress and strain fields are commonly used to assess failure. Local models are based on elastic–plastic calculations in conjunction with a model for a given physical mechanism at a point leading to fracture (Beremin, 1983). In a simple case, a post-processing treatment can be used to derive the crack initiation criteria. Fully coupled models, in contrast, are based on continuum damage mechanics or porous metal plasticity and account for the softening effect induced by cavity growth in the case of ductile failure. These models use internal variables to quantify the amount of damage at a material point. Evolution equations for the damage variables must be established; hence this class of models is computationally expensive.

In the authors’ previous studies, no clear evidence for a void growth mechanism in the magnesium alloy AZ31 could be found (Steglich and Morgeneyer, 2013). Failure in both alloys investigated here is initiated by a separation on slanted planes perpendicular to the main loading direction without any significant necking, see Fig. 2. This is observed for both orientations, RD and TD. Fracture surfaces did not show the typical structure with dimples originating from void nucleation at second phase particles followed by void growth. These results are in line with investigation using X-ray tomography (Ray and Wilkinson, 2016): In the AZ31 alloy investigated by (Ray and Wilkinson, 2016), the damage primarily manifested itself in the form of ductile dimples and twin-related microcracks. In a ZEK100 alloy (which is similar to ZE10), contraction twins are reported to form penny-shaped cracks. The ultimate fracture is due to the linkage of these twin-induced micro-cracks and is quasi-brittle in nature. A similar mechanism of quasi-brittle failure initiated by contraction twins has been reported for thicker AZ31 material (Kondori and Benzerga, 2014b). It may be worth mentioning that a significant amount of plastic deformation precedes the onset of failure. Failure strains between 10% and 20% are common for technical magnesium alloys, as for the materials under investigation here. Figure 4a shows the fracture surface of a tested tensile sample (TD orientation) of AZ31 showing facets surrounded by shallow dimples originating from micro-plasticity. Figure 4b displays the
Uncoupled approaches are suited for cases in which crack initiation and propagation are almost equivalent, and crack arresting mechanisms are absent, which is the case here. Hence, the authors will utilize local (uncoupled) criteria to predict failure as a function of the loading history.

A simple approach to modeling ductile fracture is to assume that fracture occurs when the accumulated effective strain weighted by a factor of $D(\sigma)$ reaches a critical value $D_{\text{crit}}$, that is, the critical damage indicator

$$D_{\text{crit}} = \int_{\bar{\varepsilon}^f}^{\bar{\varepsilon}} D(\sigma) d\bar{\varepsilon}$$  \hspace{1cm} (3)

Here, $\bar{\varepsilon}^f$ indicates the effective strain at fracture. The weighting factor $D$ can be set to unity, provided the effective strain is the only measure of damage, or $D$ can be taken as a function of the stress state, $D(\sigma)$. For instance, it can be defined as the largest principal stress $\sigma_1$ ($\sigma_1 > \sigma_2 > \sigma_3$) according to Cockcroft and Latham (1968) or as the effective stress. In the latter case, the dissipated plastic work is used as the damage indicator. The effective strain increment described in Eq. (3) should be carefully defined when the two-yield surface model is used, because there are two independently evolving effective strains, $\bar{\varepsilon}_g$ and $\bar{\varepsilon}_t$.

Experimental findings in which correlations between texture evolution and the event of failure were established support this assumption. Different to the mechanisms in cubic metals, the failure strain of magnesium alloys is primarily limited by a saturation of dislocation mobility. Once active slip systems underwent strain hardening, and additional deformation systems cannot be activated, further deformation can only be accommodated by mechanical twinning. For tensile loading situations causing a reduction of the sheet’s thickness, the respective mechanism is on the (10-11)-plane. Compressive loadings with positive thickness strain cause twinning on the (10-12)-plane in an early stage of deformation, not leading to immediate failure. (10-12)-twins in the post-necking microstructure of tensile samples are documented by Ray (Ray and Wilkinson, 2016) for similar material systems than in the current investigation. It is furthermore observed that damage takes the form of micro cracks that develop at twin boundaries (Kondori and Benzerga, 2014b; Ray and Wilkinson, 2016). Hence, failure seems to be related to the limits of glide, which are assumed here to coincide with the onset of (10-12)-twinning in case of in-plane tensile loading of textured (rolled) sheets.
Taking the glide effective strain $\bar{\varepsilon}_g$ for damage accumulation, the damage indicators based on the effective strain (EF), Cockcroft-Latham (CL) and plastic work (PW) can be expressed as

\begin{align*}
D_{EF} &= \int_{\bar{\varepsilon} = \bar{\varepsilon}_f}^{\bar{\varepsilon} = \bar{\varepsilon}_f} d\bar{\varepsilon}_g \quad (4) \\
D_{CL} &= \int_{\bar{\varepsilon} = \bar{\varepsilon}_f}^{\bar{\varepsilon} = \bar{\varepsilon}_f} \sigma_1 d\bar{\varepsilon}_g \quad (5) \\
D_{PW} &= \int_{\bar{\varepsilon} = \bar{\varepsilon}_f}^{\bar{\varepsilon} = \bar{\varepsilon}_f} \tilde{\sigma}_g d\bar{\varepsilon}_g \quad (6)
\end{align*}

The critical values $D_{EF}$, $D_{CL}$ and $D_{PW}$ must be determined for each material using valid experimental data. In the present study, these values were determined using a hybrid method, interpreting both the experimental data and finite element simulation data. For this purpose, the uniaxial tension of AZ31 and ZE10 sheets was simulated using a FE software package Z-Set (Besson et al., 1998). A single solid element was used and tension was applied along the RD, which was chosen as the reference orientation. The integrals on the right-hand sides of Eqs. (4-6) were calculated by post-processing the simulation results. Note that three fracture limits were obtained from the three repeated experiments for each material, as shown in Figs. 1(a) and 2(a). Those for ZE10 (RD) exhibit minimal scatter and appear superimposed on each other. The critical values $D_{EF}$, $D_{CL}$ and $D_{PW}$ were obtained by reading the integrated values at these fracture limits and then taking the average values, as listed in Table 1.

3.3. Mohr-Coulomb-type fracture criteria

Many different forms of the weighting factor $D(\sigma)$ in Eq. (3) have been suggested, so that the influence of the stress can be included in the ductile failure. Some approaches assumed the nucleation and growth of voids as the major mechanism affecting fracture and introduced the hydrostatic stress term to the damage indicator (Hancock and Mackenzie, 1976; Johnson and Cook, 1985; Oyane et al., 1980). Given that the fracture surface appearances of magnesium alloys conflict with the void growth mechanism, the present study uses another approach that is based on the Mohr-Coulomb fracture criterion, which was originally suggested for brittle materials but recently modified for ductile metals by Bai and Wierzbicki (2010). The latter will be referred to as the modified Mohr-Coulomb (MMC) criterion throughout in this paper, and is reviewed below.
The Mohr-Coulomb criterion states that fracture occurs when the sum of the normal and shear stresses reaches a critical value:

\[(\tau + c_1 \sigma_n) = c_2\]  \hspace{1cm} (7)

where \(\sigma_n\) and \(\tau\) are, respectively, the magnitudes of the normal and shear stresses on the fracture plane, and \(c_1\) and \(c_2\) are the model parameters. Instead of the terms \(\tau\) and \(\sigma_n\), the above equation can be expressed in terms of the stress invariants. Bai and Wierzbicki (2010) used the von Mises equivalent stress, triaxiality and Lode angle to replace \(\tau\) and \(\sigma_n\). The von Mises equivalent stress \(\bar{\sigma}_{vm}\) is directly related to the second invariant of stress deviator \(J_2\):

\[
\bar{\sigma}_{vm} = \sqrt{3J_2} = \frac{3}{\sqrt{2}} \mathbf{s} \cdot \mathbf{s} = \frac{1}{\sqrt{2}} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] \hspace{1cm} (8)
\]

where \(\mathbf{s}\) denotes the stress deviator. (Note that the von Mises equivalent stress is used here as a substitute to the stress invariant \(J_2\), which characterizes the stress state with the other two invariants. It is not related to the yield criterion considered in the present study.) The triaxiality \(\eta\) is defined as the mean stress \(\sigma_m\), normalized by the von Mises equivalent stress

\[
\eta = \frac{\sigma_m}{\bar{\sigma}_{vm}} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3\bar{\sigma}_{vm}} \hspace{1cm} (9)
\]

The Lode angle \(\theta\) is related to the third invariant of the stress deviator \(J_3\), normalized by the von Mises equivalent stress

\[
\cos(3\theta) = \frac{27}{2} \frac{J_3}{\bar{\sigma}_{vm}^3} = \frac{27}{2} \frac{(\sigma_1 - \sigma_m)(\sigma_2 - \sigma_m)(\sigma_3 - \sigma_m)}{\bar{\sigma}_{vm}^3} \hspace{1cm} (10)
\]

It is sometimes more convenient to use the Lode angle parameter \(\tilde{\theta}\) instead of \(\theta\)

\[
\tilde{\theta} = 1 - \frac{6\theta}{\pi} \hspace{1cm} (11)
\]

The value of the above lies in a range of \(-1 \leq \tilde{\theta} \leq 1\).
Replacing $\tau$ and $\sigma_n$ by $\bar{\sigma}_{vm}$, $\eta$ and $\bar{\theta}$, Eq. (7) can be rewritten as

$$\bar{\sigma}_{vm} = c_2 \left[ \sqrt{\frac{1 + c_i^2}{3}} \cos \left( \frac{\bar{\theta} \pi}{6} \right) + c_1 \left( \eta + \frac{1}{3} \sin \left( \frac{\bar{\theta} \pi}{6} \right) \right) \right]^{-1}$$

(12)

Approximating the hardening law in Eq. (A.13) to a power-law rule between the von Mises stress and effective strain, i.e., $\bar{\sigma}_{vm} = A \bar{\varepsilon}^n$, and multiplying the term that describes the Lode angle dependency by an additional parameter $c_3$ (Bai and Wierzbicki, 2008)

$$\bar{\sigma}_{vm} = A \bar{\varepsilon}^n \left[ c_2 + \frac{\sqrt{3}}{2 - \sqrt{3}} (1 - c_3) \left( \sec \left( \frac{\bar{\theta} \pi}{6} \right) - 1 \right) \right]$$

(13)

Note that $c_3 = 1$ reduces the Eq. (13) to $\bar{\sigma}_{vm} = A \bar{\varepsilon}^n$ and $c_3 = \sqrt{3}/2$ to $\bar{\sigma}_{tr} = A \bar{\varepsilon}^n$, where $\bar{\sigma}_{tr}$ is the Tresca equivalent stress. The introduction of the parameter $c_3$ allows more flexibility of the MMC criterion to capture the fracture behavior with respect to the Lode angle, but it also has a disadvantage that the description of strain hardening becomes inconsistent between the deformation and fracture models, as pointed out in Mohr and Marcadet (2015). It should be noted that this kind of inconsistency is unavoidable in the present study, adopting the two-surface concept for the deformation model.

Combining Eqs. (12) and (13) gives:

$$\bar{\varepsilon}^f = \left\{ c_2 \left[ c_3 + \frac{\sqrt{3}(1 - c_3)}{2 - \sqrt{3}} \left( \sec \left( \frac{\bar{\theta} \pi}{6} \right) - 1 \right) \right] \left[ \sqrt{\frac{1 + c_i^2}{3}} \cos \left( \frac{\bar{\theta} \pi}{6} \right) + c_1 \left( \eta + \frac{1}{3} \sin \left( \frac{\bar{\theta} \pi}{6} \right) \right) \right] \right\}^{-\frac{1}{n}}$$

(14)

In the above derivation, the parameter $A/c_2$ was replaced by $c_2$. The last expression can be used to derive the weighting factor $D(\sigma)$. It is assumed that the accumulated damage indicator equals 1 when fracture occurs.
Note that – different from the original model – the increment of glide strain of the deformation is considered here rather than the total strain increment.

In the case $D'$ being a function of the stress ratios, $\int D'(\sigma) d\bar{\epsilon}_g = \bar{\epsilon}_f D'(\sigma) = 1$ and therefore

$$D'(\sigma) = \left\{ c_2 \left[ c_3 + \frac{\sqrt{3}(1 - c_3)}{2 - \sqrt{3}} \left( \sec\left( \frac{\bar{\theta}\pi}{6} \right) - 1 \right) \right] \left[ \sqrt{\frac{1 + c_1^2}{3}} \cos\left( \frac{\bar{\theta}\pi}{6} \right) + c_1 \left( \eta + \frac{1}{3} \sin\left( \frac{\bar{\theta}\pi}{6} \right) \right) \right]\right\}^{\frac{1}{n}}$$

This final expression was suggested by Luo et al. (2012) and was obtained by reducing the number of parameters in the original Bai-Wierzbicki criterion ($c_{\eta}^x = 1$ and $c_\eta = 0$). It is recommended that the parameter $n$ be optimized using the hardening curve, such that the three remaining parameters $c_1 - c_3$ can be determined using fracture tests with different loading conditions. The parameter $c_1$ controls the contribution of the normal stress to the fracture and $c_2$ tunes the magnitude of the critical stress, as implied by Eq. (7). Furthermore, $c_3$ controls the influence of the Lode angle dependency, as implied by Eq. (13).

The MMC criterion essentially assumes that failure is isotropic. For instance, if uniaxial tension is applied along the RD and TD of a given material, Eq. (16) yields identical values for both directions and, consequently, Eq. (15) predicts that fracture would occur at the same amount of effective strain in both cases. This might be a valid assumption for AZ31 but not for ZE10, for which the fracture limit significantly depends on the loading direction, as shown in Fig. 2(a). For this reason, the extended MMC criterion suggested in (Luo et al., 2012) is additionally considered for ZE10. This extended model is referred to as the EMMC criterion to distinguish it from MMC. The EMMC criterion uses another definition for the effective strain increment, introducing a linearly transformed plastic strain tensor

$$d\bar{\epsilon}' = \frac{2}{3} B d\epsilon^p_g : B d\epsilon^p_g$$

where $B$ is the fourth-order transformation tensor and can be expressed as
The anisotropic fracture behavior is then described by using $\bar{d}\varepsilon'$ instead of $\bar{d}\varepsilon$ in Eq. (15). Therefore, the EMMC criterion involves total six parameters: $\beta_{22}, \beta_{33}$ and $\beta_{12}$ in addition to $c_1 - c_3$. Note that the use of the von Mises equivalent stress suggests using the failure criterion for isotropic materials only. In line with many other applications, its use here together with a model of anisotropic plasticity is purely phenomenological and does not guarantee that plasticity model and damage indicator are “associative”. This extends to the transformation given by $B$, which is different from the linear transforms used in the yield function.

The parameters of the MMC or EMMC criteria can be determined using the stress-strain history of a material up to fracture under different stress states. The complete stress-strain history of a material is usually not available from the experiments and can be estimated using FE simulations, as described in the following subsection. Note that only the contribution of the glide strain $\varepsilon_g^P$ is considered for $d\varepsilon'$ in the two-yield surface model, as implied by Eq. (17), which differs from the original EMMC model which considers the overall effective strain.

While the constitutive model and the failure criteria are formulated for general three-dimensional stress states, it will be applied here to 3D solid finite elements and shell elements. Shell elements are based on the plane stress condition, which imposes a dependency between the triaxiality and Lode angle parameter through the following relationship

$$\cos\left(\frac{\pi}{2}(1 - \bar{\theta})\right) = -\frac{27}{2} \eta\left(\eta^2 - \frac{1}{3}\right)$$

(19)

### 3.4. Parameter optimization for Mohr-Coulomb-type fracture criteria

The three parameters of the MMC criterion were optimized so that the fracture limits observed in UT, NB and SH tests along the RD lead to the critical value of 1 ($D_{MMC} = 1$). The six parameters of the EMMC criterion were similarly optimized using the fracture limits observed in UT, NB and SH tests along both the RD and TD. The experiment-simulation hybrid method is adopted again for the parameter calibration. FE simulations for UT, NB and SH tests were conducted with solid elements, thus taking into account the three dimensional
stress state. First, uniaxial tension tests were approximated by single element simulations. The stress state was therefore uniform with constant triaxiality $\eta = 1/3$ and Lode angle parameter $\tilde{\theta} = 1$. Fracture occurs when the engineering strain reaches the limit indicated in Figs. 1(a) and 2(a). The NB and SH simulations were conducted using FE models that were constructed based on the actual experiments. As shown in Fig. 5, only an eighth of the notched specimen was modeled considering the three-fold symmetry and the entire geometry was modeled for the shear specimen. It was assumed that fracture occurs when the NOD or pin distance reaches the fracture limits indicated in Figs. 1 and 2.

In the application of local damage models, stress and strain histories have to be integrated over the loading history. For the calibration of such model it is necessary to identify the fracture initiation location. With this information, the critical state expressed via the damage indicator can be defined. While this is straightforward in the case of ductile metals with strain localization prior to failure, it was not possible in case of the two magnesium sheets considered in the present investigation. Excluding the possibility of edge fracture initiation, it was consistently assumed for both materials and orientations in the case of the shear samples that failure initiation is in the center of the sample (Luo et al., 2012), close to the position of maximum strain (Roth and Mohr, 2014). This treat appeared not to be justified in case of the NB. Here, it was assumed that failure initiates at the peak of triaxiality, which occurs near the outer edge in the ligament away from the notch tip.

As an example, the distribution of triaxiality along the centerline predicted for the NB specimen at the fracture point is shown in Figs. 6 and 7. The figures reveal that the triaxiality, in most cases, the maximum triaxiality occurs near the outer edge, at a distance about 1 mm from the notch tip. An exception is found for ZE10 along the RD, as shown in Fig. 7(a), where the triaxiality is maximum in the center of the specimen. Even in this case, the local maximum near the edge was selected as the site of fracture initiation for consistency with the other cases. A similar plot for the Lode angle parameter is superimposed in the same figure (red dotted curves) and the data point at the location of maximum triaxiality is marked as symbols in all of the plots. As expected, the Lode angle parameter extracted from the 3D finite element simulation differs from the one calculated according Eq. 19 from the triaxiality (plane stress case, dashed line in Figs. 6-9). This indicates that a 3D stress state evolves at the site of the assumed crack initiation. The distributions of the effective strains, $\tilde{\varepsilon}_g$ and $\tilde{\varepsilon}_t$, are also shown in Figs. 6 and 7, suggesting that the deformation in the center of the NB specimen is mostly achieved by the dislocation glide.
Figures 8 and 9 show the distributions of the triaxiality, Lode angle parameter and effective strain along the centerline of the shear specimen. Since the entire geometry of the shear specimen was considered here, the distributions are nearly symmetric with respect to the center of the specimen. The distributions of the effective strain suggest that the deformation in the shear specimen is accommodated by both dislocation glide and twinning, and the contribution of twinning is more significant than that of glide in the case of ZE10.

The evolutions of triaxiality and Lode angle parameter of the critical elements in the UT, NB and SH simulations are shown in Figs. 10 and 11 for AZ31 and ZE10, respectively. The experimentally obtained failure points are included. Only the RD results are shown in these figures, but similar trends are found in the TD results. The constant values of $\eta = 1/3$ and $\theta = 1$ are reported for the UT simulation because a single element was used in this case. The deformation is not strictly proportional for the other cases. The triaxiality of the critical element is maintained for both the NB and SH cases, but the Lode angle parameter shows visible fluctuations throughout the deformation, particularly for the SH case. The integration in Eqs. (15-16) was performed using the instantaneous triaxiality and Lode angle parameter at each increment and, therefore, the evolutions of the stress state were taken into account in the calculation.

The MMC and EMMC parameters were determined using an iterative procedure (the simplex method) so that the accumulated damage indicator reaches 1 at the fracture limits. The parameters are provided in Table 2. Note that the exponent $n$ was independently obtained by power-law fitting of the uniaxial tension curve and excluded from the optimization procedure.

From these experimental/numerical data, a chart is established, evidencing the dependencies of the local failure strain on the triaxiality and the Lode angle parameter for both materials in their RD orientation. This dependency based on data obtained from tests (UT, NB and SH) along the RD of both materials is shown in Fig. 12, revealing the failure surfaces for AZ31 and ZE10, respectively. By arbitrarily selecting a triaxiality of 0.33, Fig. 12(a), and the Lode angle parameter of 0.5, Fig. 12(b), the intersection lines of the respective planes with the failure surfaces reveal possible trends. Considering a constant triaxiality of 0.33, Fig. 12(a), for AZ31 the fracture strain is increasing with increasing Lode angle parameter, but it is decreasing in case of ZE10. For a constant Lode angle parameter of 0.5, Fig. 12(b), both materials show a decrease of failure strain with increasing triaxiality, as expected.

As already mentioned, the above assessment and the following analysis assume a distinct failure initiation point. Local failure strains and failure model parameters are determined based on this choice. This
assumption may appear somehow arbitrary, as other criteria (maximum strain, maximum Lode angle parameter) would be possible as well. However, from Figs. 6-9 it becomes clear that possible changes do not alter the general trends of the failure loci shown in Fig. 12.

3.5. Comparison of fracture criteria for different stress states

In the previous sections, the parameters of the three simple fracture criteria and two Mohr-Coulomb-type criteria were determined for AZ31 and ZE10, as summarized in Tables 1 and 2. Using the complete stress-strain history of the critical elements obtained in the UT, NB and SH simulations, the fracture criteria were evaluated for these loading cases, both along the RD and TD. The results for AZ31 and ZE10 are presented in Figs. 13 and 14, respectively, where the integrated damage indicators of the criteria were normalized by their critical values, $D_{EF}$, $D_{CL}$ and $D_{PW}$, obtained in uniaxial tension along the RD (Table 1). Therefore, the corresponding data (tension along the RD) are always equal to unity for the isotropic criteria. For the MMC and EMMC criteria, the input data used for the parameter calibration are indicated with solid circles.

The results for AZ31 reveal that none of the isotropic criteria can simultaneously capture the fracture limits in the three loading cases. When their critical values ($D_{EF}$, $D_{CL}$ and $D_{PW}$) are determined based on UT, these criteria predicted delayed fracture in the SH and NB cases. The MMC criterion can capture the fracture limits in all three cases. The MMC parameters obtained from the RD data provide reasonable predictions for the TD cases, because the fracture behavior of AZ31 is similar along the RD and TD. The results for ZE10 are very different from those for AZ31 in that the isotropic criteria predict delayed fracture in the SH and early fracture in NB. Moreover, the MMC criterion was not able to simultaneously capture the fracture limits in the three loading cases, as shown in Fig. 14(d). (Note that different optimization schemes and sets of initial values were examined but none produced better results). Moreover, ZE10 exhibits strong anisotropy in the fracture limits between the RD and TD, and this anisotropy cannot be captured by the MMC criterion. Only the EMMC could capture the fracture limits in all three loading cases, along both the RD and TD, as shown in Fig. 14(e). These results suggest that the studied isotropic fracture criteria are not sufficient to characterize the fracture limits of AZ31 and ZE10 under a wide range of stress states, and even the MMC criterion may not be able to capture the fracture limits for some materials, such as ZE10. Also, if the fracture behavior exhibits strong anisotropy, only EMMC can provide a reasonable description of the fracture behavior among the studied criteria.

4. Structure assessment
4.1. Square tube compression

The calibrated fracture criteria were used to predict the failure of square tubes under two loading conditions: axial compression and three-point bending. Hollow tubes with a square cross-section were prepared by cutting and welding AZ31 and ZE10 sheets in two orientations, i.e., the axial direction parallel to either the RD or TD of the sheets (Steglich et al., 2015). The dimensions of the tube structures were approximately 50 mm×50 mm×400 mm. For the compression tests, the two ends (50 mm×50 mm×40 mm, each) of the tube were fixed in the grips of the testing machine, see Fig. 15(a). The structure was then subjected to quasi-static axial compressive loading until fracture, which always initiated near the edge of the tube. The axial load and end displacement were recorded throughout the tests and are shown as dotted lines in Figs. 16 and 17 for AZ31 and ZE10, respectively.

The tube compression simulations were conducted using the commercial Abaqus/Standard FE program (implicit code) with a user-material subroutine for the two-yield surface model linked with the Z-set via its utility Z-mat. Only the deformable section of the structure (50 mm×50 mm×320 mm) was modeled and the displacement-based boundary condition was applied for compressive loading. Shell elements were used in these simulations because they are computationally more efficient and it is also common to use them in this kind of structural analysis.

The simulation results were post-processed to calculate the damage indicators. An example is shown in Fig. 18(a) for the MMC criterion. The largest damage accumulation was found near the edge for most of the considered criteria, except for the EF criterion in the case of ZE10 (RD), for which it was found approximately 10-15 mm from the edge. The predicted failure limits are indicated on the predicted force-displacement curves in Figs. 16 and 17 for AZ31 and ZE10, respectively. Here, the force was normalized by the cross-sectional area of the tube. For AZ31, the predicted force-displacement curves are in good agreement with the experimental data, with a plateau followed by an increase in the load. This degree of agreement is remarkable in comparison with the prediction using the isotropic von Mises model based on the tensile test results, plotted as a red curve in Fig. 16(a). The predicted fracture limits are at around the displacement of 35 mm, slightly later than the actual fracture limits. Nevertheless, considering the experimental scatter presented in Fig. 16(a), the fracture criteria seem to provide reasonable predictions, at least for the RD case. Next, for ZE10, the load-displacement is also accurately predicted, capturing the initial peak followed by a plateau that continues until fracture, as shown in Fig. 17. The predicted fracture limits are at around 30 mm, very close
to the actual fracture limits for both the RD and TD cases. Also for ZE10, the von Mises model produces a highly inaccurate force-displacement response.

In summary, the force-displacement responses are accurately predicted for the tube compression of AZ31 and ZE10, thus validating the capability of the two-yield surface model, and the fracture limits are reasonably predicted in most cases. However, all the studied fracture criteria lead to very similar failure predictions in this example, even though some do not accurately capture the fracture behavior under stress states other than uniaxial tension, as discussed in Figs. 13 and 14. A possible explanation for this is that, once the tube structure reaches a certain instability state, additional plastic deformation is localized and damage rapidly accumulates and reaches the critical value regardless of the criterion.

4.2. Three-point bending of square tubes

Three-point bending tests were conducted for AZ31 and ZE10 square tubes having the same fabrication parameters and dimensions as those described above. The cross-head velocity (10 mm/min) was adjusted to ensure quasi-static conditions. The roller supports (320-mm wide) had a diameter of 40 mm each, and the cylindrical punch was 30 mm in diameter, see Fig 15(b). For each material, two orientations were tested: one with the RD along the axial direction of the profile, and the other with the TD along the axial direction. The load was applied under displacement control. The current punch load was normalized by the net cross-section of the profile (360 mm²) as shown in Figs. 19 and 20. For AZ31, the decrease in the load at a punch displacement of 11 mm was related to the link-up of smaller cracks that had already nucleated. Discrete load drops corresponding to the nucleation of macroscopic cracks are not observed in the case of ZE10. It was therefore difficult to judge, based on these data, when fracture initiated. Visual inspection was also problematic because the tested specimens revealed cracks on the upper edges of the tube, which is in contact with the punch and thus cannot be observed during the test. A rational anticipation in the case of AZ31 is that, in the experiments, crack initiation occurs close to the maximum load achieved during the test. In the case of ZE10, crack initiation occurs in the experiment after maximum load was achieved. Fig. 21 shows the fractured samples after the three-point bending tests of AZ31 (a) and ZE10 (b). By comparing the two cracking scenarios, it becomes clear why the force-displacement records of the two materials are different. In the case of AZ31, cracks propagate in axial direction and along the side. For ZE10, only one crack propagates at each side in axial direction while bulged shoulders of sheet metal are formed on the vertical sections. These shoulders act as rests and contribute to the stiffness of the structure, hence "hiding" the cracks in the force-displacement record.
FE simulations of the three-point bending were conducted using Abaqus/Standard, in which the tube was meshed using shell elements and the tools were modeled using analytical rigid surfaces. The one-fold symmetry of the problem was exploited. The simulation results were then post-processed to calculate the damage indicators. As shown in Fig. 18(b), the largest damage accumulation was found near the upper edge (the punch is not shown in the figure), which is consistent with the experiment results. The predicted load-displacement data are in good agreement with the measured data at the beginning of the process, as shown in Figs. 19 and 20, again demonstrating the viability of the two-yield surface model. The measured and predicted curves start to diverge at a punch displacement of approximately 6 mm. It is likely that cracks have initiated at this stage. Under this assumption, the MMC criterion seems to provide the most reasonable prediction for AZ31 and all the criteria except EMMC for ZE10. It is surprising that the EMMC criterion predicts extremely delayed fracture in this example, although it perfectly captures the fracture behaviors in the basic mechanical tests, as shown in Fig. 14. This will be further discussed in the following subsection, emphasizing three interrelated aspects.

4.3 Discussion

Limitations in the optimization of fracture criteria: As shown in Fig. 18, the largest damage accumulation was usually found near the edge in the tube-compression and three-point bending simulations. Two representative finite elements, 'TOP' and 'SIDE,' were selected in the critical region, as indicated in the figure, and the stress history of these elements was extracted for further examination. Because shell elements were used in these simulations, the stress states of these elements evolve following the plane stress condition, as given by Eq. (19), and which is plotted as a black dotted curve in Fig. 22. This figure shows that the stress states of both the TOP and SIDE elements are maintained mostly in the region of negative triaxiality in the case of tube compression. For three-point bending, the TOP and SIDE elements follow distinct stress paths in that the former experiences positive triaxiality while the latter experiences negative triaxiality. It is, therefore, highly probable that the failure of a tube structure initiates in the region of negative triaxiality. Hence, an elementary characterization of the failure of thin sheets under compressive stress is desirable. However, it is not practical to measure the failure behavior of thin sheets under compressive stress. For instance, as observed in the present study, the fracture limits in uniaxial compression could not be determined because of the combined effects of buckling, partial debonding of the glued sheets, and fracture. For this reason, the parameters of the fracture criteria were optimized based solely on the UT, NB, and SH data, all of which correspond to zero or positive triaxiality. It may be possible to further improve the failure
prediction by directly measuring the fracture limits in compressive stress and then including these data in
the calibration of fracture criteria, although it would require advanced experimental techniques to avoid the
occurrence of buckling and the like which could lead to instability before fracture.

**Assessment of failure criteria under compressive stress:** As observed in Section 4, the MMC criterion tends to
predict an earlier failure than the other criteria for tube compression and three-point bending. This can be
rationalized, given that the relevant position is in the region of negative triaxiality. In this regime, most of the
strain is due to mechanical twinning. In the concept pursued here, this strain contribution is assumed to have
no effect on the damage accumulation. Therefore, the glide strain is highly asymmetrical in tension and
compression. For example, Fig. 23 compares the damage indicators in uniaxial compression for AZ31 up to
the point of maximum stress observed in Fig. 1(b). (Note that the point of maximum compressive stress does
not correspond to the actual fracture limit, but it is used as the reference state to qualitatively compare the
damage accumulation.) In the same way as in Fig. 13, the damage indicators were normalized by the values
listed in Table 1 and the MMC damage indicator was calculated using the parameters listed in Table 2. For
AZ31, the MMC criterion results in more rapid accumulation of damage than the other criteria in uniaxial
compression. Because the stress state is also compressive in tube compression and three-point bending, the
MMC criterion is likely to predict the fastest damage accumulation in these cases. This statement is also
supported by Fig. 13, in which the MMC criterion predicts the largest damage indicator under shear stress
(or zero-stress triaxiality). This can explain why the MMC criterion predicts the earliest failure for AZ31 in
the above two examples, see Figs. 16 and 19. Given that the MMC criterion produces the most reasonable
predictions for both AZ31 and ZE10, it appears to produce an appropriate description of the failure behavior
in the region of negative triaxiality, even though the parameters are optimized with the data measured under
positive triaxiality.

**Comparison of AZ31 and ZE10 in three-point bending:** In the case of three-point bending, the maximum
damage accumulation was predicted on the inner surface of the profile on the 'TOP' side (see Fig. 18). Here,
the stress triaxiality is positive (it is negative on the outer surface due to predominant bending). Interestingly,
the evolution of the stress triaxiality as a function of the loading differs between the AZ31 and ZE10. This
difference is primarily due to the load-carrying capacity of the side parts of the profiles, which is strongly
influenced by the strength differential effect of the materials. Hence, the deformed cross-sections of AZ31
and ZE10 for a given punch displacement appear different, as shown in Fig. 24. The differences in cross-
section deformation between the two materials are also visible in Fig. 21. Figure 25(a) shows the evolution
of triaxiality and Lode angle parameter at the inner surface of the TOP element in three-point bending. Note
that both stress invariants are considered in the calculation of the evolution of the damage indicators for the MMC and EMMC criteria using Eq. (16). The evolution of the damage parameters is visualized for the MMC criterion in Fig. 25(b), which shows that the damage indicator accumulates faster for ZE10 than for AZ31, leading to earlier crack initiation with ZE10. This effect is amplified by the evolution of the effective glide strain at this position: At a given displacement, greater deformation due to dislocation glide is predicted at this position in the case of ZE10, relative to AZ31, as shown in Fig. 25(b). This indicates that the strain is more localized for ZE10 than for AZ31, which explains the unexpected difference in the failure prediction for three-point bending, as shown in Figs. 19 (AZ31) and 20 (ZE10). This effect is further amplified when the EMMC criterion is used, leading to an unrealistic prediction with a very large failure displacement, as shown in Fig. 20(a).

5. Conclusions

In the present study, three simple isotropic failure criteria and two Mohr-Coulomb-type fracture criteria were examined and applied to the prediction of the fracture limits of AZ31 and ZE10 magnesium alloys under various stress states. The parameters of the criteria were identified using the uniaxial tension data, and those of the Mohr-Coulomb-type criteria were identified using both tension-based and shear data, covering a wider range of stress triaxiality and Lode angle. An anisotropic hardening model was used with the two-yield surface formulation to represent the tension-compression asymmetry in yielding and hardening of the two magnesium alloys. This model features two distinct deformation mechanisms, namely, dislocation glide and deformation twinning, under different loading conditions. It was assumed that only the glide mechanism contributes to damage accumulation for all the criteria that were considered. The fracture criteria were evaluated by predicting the failure of square tubes under axial compression and three-point bending. The coupling of the experimental and numerical analysis led to the following conclusions:

- Commonly, the total effective strain is used as the kinematic quantity in the case of ductile failure models. While this has proven being capable of producing substantiated predictions in the case of symmetrical materials (Lou et al., 2014; Mohr and Marcadet, 2015), in the cases considered in the present study, the respective predictions were defective. In the structural assessments based on shell elements, failure was systematically predicted too early. Given that twinning is mainly activated in a compressive stress state, in which fracture is generally suppressed, it was decided to consider only the glide effective strain for failure prediction in the present study.
• The considered isotropic criteria could not capture the fracture limits of AZ31 and ZE10 in shear and U-notched tension when the parameters were identified based on uniaxial tension. On the other hand, the MMC and EMMC criteria could reproduce the measured fracture limits in all three loading cases for the specimens along the same loading direction (RD).

• The MMC criterion was not sufficient to capture the fracture limits of ZE10 along the TD due to the strong anisotropy in failure strain of ZE10. Among the considered criteria, only the EMMC model could capture the fracture limits along both the RD and TD in all three loading cases. This is more of a consequence of its flexibility, originating from linear transforms, than a mapping of the physical failure mechanisms.

• The two-yield surface model accurately predicted the force–displacement responses of square tubes under tube compression and three-point bending. On the contrary, the isotropic von Mises model resulted in highly inaccurate force–displacement predictions and should therefore not be used.

• For failure prediction in tube compression, all the considered fracture criteria predicted the fracture limits with reasonable accuracy for AZ31 and ZE10 along both the RD and TD. For three-point bending, the MMC criterion provided the best prediction for AZ31 if the divergence points between the measured and predicted load–displacement curves are interpreted as the initiation of fracture. However, the EMMC criterion predicted a delayed fracture for ZE10, relative to the other criteria, even though it perfectly captured the fracture behaviors under the tension-based and shear-loading conditions. This is attributed to the strong localization of deformation in the critical region of the tube structure. The authors can therefore confirm the statement already made (Luo et al., 2012), that this criterion has lost its original physical meaning and should therefore be used with caution.

• The discrepancy between the measured and predicted failure limits can be explained considering the stress history in the critical regions. The stress triaxiality in the critical regions was mostly negative for tube compression and mixed positive–negative for three-point bending. Therefore, the considered fracture criteria, which were calibrated using the stress–strain data under positive triaxiality, may not accurately represent the failure behaviors in the region of negative triaxiality. In the same context, the difference in the failure predictions between the isotropic and Mohr-Coulomb-type criteria may also be attributed to the different predictions for the stress state with negative triaxiality.
• The deformation behavior of the two magnesium alloys considered show significant differences. Their respective anisotropy and the evolution of the strength differential effect lead to varying deformation patterns in the crushing and three-point bending experiments. As demonstrated in the case of three-point bending, the localization of deformation is more pronounced in the case of the profile fabricated from ZE10 rather than its counterpart made of AZ31. This directly affects the global load response, and indirectly determines the relationship between the instability point (maximum load) and the onset of failure. Both states coincide for AZ31, but differ significantly for ZE10.

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Appendix A. Two-yield surface model for Mg alloys

Assuming the additive decomposition of the strain increment, the total strain increment consists of the elastic and plastic parts

\[ d\varepsilon = d\varepsilon^e + d\varepsilon^p \]  \hspace{1cm} (A.1)

The bold letters indicate second-order tensors unless otherwise specified. The elastic strain is determined using the conventional isotropic Hooke’s law with Young’s modulus \( E = 43 \) GPa and Poisson’s ratio \( \nu = 0.3 \). The plastic strain increment is further decomposed into that caused by dislocation glide, \( d\varepsilon^p_g \), and that caused by twinning, \( d\varepsilon^p_t \)

\[ d\varepsilon^p = d\varepsilon^p_g + d\varepsilon^p_t \]  \hspace{1cm} (A.2)
First, the deformation by dislocation glide is described using the non-quadratic, anisotropic yield criterion proposed by Barlat et al. (1991) in conjunction with the combined isotropic-kinematic hardening law proposed by Chaboche (2008). The yield criterion can be expressed as

$$\phi_g = f_g - \bar{\sigma}_g = \frac{1}{2} (|\beta_2 - \beta_3| a_g + |\beta_3 - \beta_1| a_g + |\beta_1 - \beta_2| a_g) \frac{1}{a_g} - \bar{\sigma}_g = 0 \quad (A.3)$$

where $\bar{\sigma}_g$ describes the isotropic hardening of the yield surface and $a_g$ is the yield function exponent. $\beta_1, \beta_2,$ and $\beta_3$ are the principal values of a linearly transformed stress tensor $\beta$ such that

$$\beta = L : B \quad (A.4)$$

Here, $B$ is the Cauchy stress from which the back stress is subtracted, $B = \sigma - X$. The fourth-order transformation tensor $L$ is expressed in the Voigt notation as

$$L = \begin{bmatrix}
\frac{(l^{LL} + l^{SS})}{3} & -\frac{l^{SS}}{3} & -\frac{l^{LL}}{3} & 0 & 0 & 0 \\
-\frac{l^{SS}}{3} & \frac{(l^{TT} + l^{TT})}{3} & -\frac{l^{TT}}{3} & 0 & 0 & 0 \\
-\frac{l^{LL}}{3} & -\frac{l^{TT}}{3} & \frac{(l^{TT} + l^{LL})}{3} & l^{TL} & 0 & 0 \\
0 & 0 & 0 & l^{LS} & 0 & 0 \\
0 & 0 & 0 & 0 & l^{LS} & 0 \\
0 & 0 & 0 & 0 & 0 & l^{ST}
\end{bmatrix} \quad (A.5)$$

where the $l^{TT}, l^{LL}, l^{SS}, l^{TL}, l^{LS}$ and $l^{ST}$ parameters describe the plastic anisotropy. The plastic strain increment caused by dislocation glide is obtained using the associated flow rule with yield surface $\phi_g$

$$d\varepsilon^{p}_g = d\lambda_g \frac{\partial f_g}{\partial B} = d\lambda_g \frac{\partial f_g}{\partial (\sigma - X)} \quad (A.6)$$

Since $f_g$ is a homogeneous function of degree one, $f_g d\lambda_g = \bar{\sigma}_g d\lambda_g = (\sigma - X) : d\varepsilon^{p}_g$, which is equal to the plastic work dissipated by dislocation glide. The plastic multiplier $d\lambda_g$ therefore corresponds to the effective strain increment achieved by dislocation glide, $d\lambda_g = d\bar{\varepsilon}_g$.

The translation of the yield surface $\phi_g$ is described by the evolution of the back stress tensor $X$. An intermediate variable $\alpha$ is introduced such that
\[ X = \frac{2}{3} C : \alpha \]  
(A.7)

and the evolution of \( \alpha \) is described as

\[ d\alpha = d\varepsilon^p_g - \frac{3}{2} \frac{d\bar{\varepsilon}}{d\varepsilon} D : X \]  
(A.8)

Here, \( C \) and \( D \) are fourth-order tensors including parameters for kinematic hardening. For simplicity \( D = D_g I \) is assumed, where \( I \) is the fourth-order identity tensor, and \( C \) has a diagonal form including six parameters

\[
C = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & c^{LL} & 0 & 0 & 0 & 0 \\
0 & 0 & c^{SS} & 0 & 0 & 0 \\
0 & 0 & 0 & c^{TL} & 0 & 0 \\
0 & 0 & 0 & 0 & c^{LS} & 0 \\
0 & 0 & 0 & 0 & 0 & c^{ST}
\end{bmatrix} \]  
(A.9)

Next, the deformation by twinning is described using the non-quadratic, anisotropic and asymmetric yield criterion proposed by Cazacu et al. (2006). This yield criterion was chosen to take account of the strength differential effect. For magnesium sheet alloys, the asymmetry of the yield surface is related to deformation twinning. The yield criterion can be expressed as

\[
\phi_t = f_t - \bar{\sigma}_t = ((|\delta_1| - k\delta_1)^{\alpha_1} + (|\delta_2| - k\delta_2)^{\alpha_2} + (|\delta_3| - k\delta_3)^{\alpha_3})^{\frac{1}{\alpha}} - \bar{\sigma}_t = 0
\]  
(A.10)

where \( \delta_1, \delta_2 \) and \( \delta_3 \) are the principal values of a linearly transformed stress deviator \( \delta \) such that

\[
\delta = M : T : \sigma
\]  
(A.11)

Here, \( T \) is the fourth-order tensor that transforms the stress tensor to its deviator and \( M \) is the fourth-order tensor including anisotropy parameters.
\[ M = \begin{bmatrix}
    m^{TT} & M^{TL} & M^{TS} & 0 & 0 & 0 \\
    M^{TL} & m^{LL} & M^{LS} & 0 & 0 & 0 \\
    M^{TS} & M^{LS} & m^{SS} & 0 & 0 & 0 \\
    0 & 0 & 0 & m^{TL} & 0 & 0 \\
    0 & 0 & 0 & 0 & m^{LS} & 0 \\
    0 & 0 & 0 & 0 & 0 & m^{ST} 
\end{bmatrix} \]  

(A.12)

where \( m^{ij} \geq 0 \) and \( M^{ij} \geq 0 \) \((i, j = T, L, S)\). Parameter \( k \) in Eq. (A.10) controls the strength differential effect \((-1 \leq k \leq 1)\). For \(-1 \leq k < 0\), the yield stress in tension is smaller than that in compression, and for \(0 < k \leq 1\) the yield stress in tension is larger than that in compression, as in the case of magnesium alloys.

The plastic strain increment caused by twinning, \( d\varepsilon^p_t \), is obtained using the associated flow rule with the yield surface \( \phi_t \). The plastic work dissipated by twinning is \( \bar{\sigma}_t d\varepsilon_t = \sigma : d\varepsilon^p_t \), where \( d\varepsilon_t \) is the effective strain increment achieved by twinning.

The interaction between two deformation mechanisms is realized through hardening of the two yield surfaces defined as follows

\[
\bar{\sigma}_g(\bar{\varepsilon}_g, \bar{\varepsilon}_t) = R_g + H_g \bar{\varepsilon}_t + Q_{1g}[1 - \exp(-b_{1g}\bar{\varepsilon}_g)] + Q_{2g}[1 - \exp(-b_{2g}\bar{\varepsilon}_g)]  
\]  

(A.13)

\[
\bar{\sigma}_t(\bar{\varepsilon}_t) = R_t + H_t \bar{\varepsilon}_t + Q_{1t}[\exp(b_{1t}\bar{\varepsilon}_t) - 1] + Q_{2t}[1 - \exp(-b_{2t}\bar{\varepsilon}_t)]  
\]  

(A.14)

Note that the isotropic hardening of the glide yield surface is governed by both \( \bar{\varepsilon}_g \) and \( \bar{\varepsilon}_t \) to address the obstacles to dislocation motion resulting from mechanical twinning. As implied in Eq. (A.14), the hardening of the twinning yield surface is not affected by the glide mechanism. In a general 3-dimensional loading scenario, the deformation model includes the following sets of parameters.

- Anisotropy parameters for yield surface \( \phi_g \): \( l^{TT}, l^{LL}, l^{SS}, l^{TL}, l^{LS} \) and \( l^{ST} \)
- Anisotropy parameters for yield surface \( \phi_t \): \( m^{TT}, m^{LL}, m^{SS}, m^{TL}, m^{LS}, M^{TT}, M^{LL} \) and \( M^{LS} \)
- Kinematic hardening parameters for yield surface \( \phi_g \): \( c^{TT}, c^{LL}, c^{SS}, c^{TL}, c^{LS}, c^{ST} \) and \( D_g \)
- Strength differential effect parameter for yield surface \( \phi_t \): \( k \)
- Yield function exponents: \( a_g \) and \( a_t \)
- Isotropic hardening parameters for yield surface \( \phi_g \): \( R_g, H_g, Q_{1g}, b_{1g}, Q_{2g} \) and \( b_{2g} \)
Isotropic hardening parameters for yield surface $\phi_t$: $R_t$, $H_t$, $Q_{1t}$, $b_{1t}$, $Q_{2t}$ and $b_{2t}$

The above parameters were calibrated for AZ31 and ZE10 sheets in a previous study (Steglich et al., 2016) so that the engineering stress-strain or force-displacement data shown in Figs. 1 and 2 can be reproduced. Since mechanical tests along the thickness direction of the thin sheets were not performed for obvious reasons, the model parameters scaling the thickness stress components remain undetermined. They are set to be equal 1 in the 3D-simulations.

For the reader’s convenience, the optimized parameters of the plasticity model for AZ31 and ZE10 sheets are given in Tables A1 and A2.

Appendix B. Failure prediction based on the total strain

In principle, the failure criteria given by Eqns. 4-6, 15, and 17 can be formulated with the total plastic strain increment $d\varepsilon_p = d\varepsilon_p^P + d\varepsilon_p^F$ rather than the plastic glide strain increment. Albeit this suggests itself, it was not used here. One (micromechanically motivated) reason is described in section 3.2. Another motive is shown in the Fig. A2, illustrating the assessment of the crushing test of ZE10 along the rolling direction. Since the early deformation in this test is accommodated by twinning, the EF criterion (calibrated from a tensile test causing glide strain) leads to an obviously unrealistic failure prediction. The other criteria suffer from the same intrinsic feature, although this is less obvious in the figure.

References


Table A1 parameters of anisotropy of the plasticity model for AZ31 and ZE10 sheets (after Steglich et al., 2016). Except $c_{ij}$ (dimension MPa), all parameters are dimensionless.

<table>
<thead>
<tr>
<th></th>
<th>AZ31</th>
<th>ZE10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l^{TT}$</td>
<td>0.898</td>
<td>0.94</td>
</tr>
<tr>
<td>$l^{LL}$</td>
<td>1.06</td>
<td>1.62</td>
</tr>
<tr>
<td>$l^{SS}$</td>
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<td>$l^{TL}$</td>
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<tr>
<td>$l^{LS}$</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$l^{ST}$</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$m^{TT}$</td>
<td>7.81</td>
<td>19.9</td>
</tr>
<tr>
<td>$m^{LL}$</td>
<td>1.41</td>
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<td>1.40</td>
<td>1.54</td>
</tr>
<tr>
<td>$m^{TS}$</td>
<td>0.013</td>
<td>0.013</td>
</tr>
<tr>
<td>$a_t$</td>
<td>0.013</td>
<td>0.013</td>
</tr>
<tr>
<td>$M^{TL}$</td>
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<td>0.013</td>
</tr>
<tr>
<td>$m^{TL}$</td>
<td>0.667</td>
<td>0.667</td>
</tr>
<tr>
<td>$m^{LS}$</td>
<td>0.57</td>
<td>1.0</td>
</tr>
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<td>$m^{ST}$</td>
<td>1.0</td>
<td>1.0</td>
</tr>
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<td>$M^{LS}$</td>
<td>121040</td>
<td>24508</td>
</tr>
<tr>
<td>$M^{TS}$</td>
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<td>1.0</td>
</tr>
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<td>$c^{TT}$</td>
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<td>19.9</td>
</tr>
<tr>
<td>$c^{LL}$</td>
<td>1.41</td>
<td>1.85</td>
</tr>
<tr>
<td>$c^{SS}$</td>
<td>1.40</td>
<td>1.54</td>
</tr>
<tr>
<td>$c^{TL}$</td>
<td>0.013</td>
<td>0.013</td>
</tr>
<tr>
<td>$c^{LS}$</td>
<td>0.013</td>
<td>0.013</td>
</tr>
<tr>
<td>$c^{ST}$</td>
<td>0.013</td>
<td>0.013</td>
</tr>
<tr>
<td>$D_g$</td>
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<td>1669</td>
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<tr>
<td>$k$</td>
<td>25004</td>
<td>0.73</td>
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Table A2 hardening parameters of the plasticity model for AZ31 and ZE10 sheets (after Steglich et al., 2016).

<table>
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<tr>
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<tr>
<td>$R_g$ [MPa]</td>
<td>145.3</td>
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<td>$H_g$ [MPa]</td>
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<td>$Q_{1g}$ [MPa]</td>
<td>178.8</td>
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<tr>
<td>$Q_{2g}$ [MPa]</td>
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<td>$R_t$ [MPa]</td>
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<td>$H_t$ [MPa]</td>
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<tr>
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<tr>
<td>$Q_{2t}$ [MPa]</td>
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<tr>
<td>$b_{1g}$</td>
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<tr>
<td>$b_{2g}$</td>
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</tr>
<tr>
<td>$b_{1t}$</td>
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<td>0.0</td>
</tr>
<tr>
<td>$b_{2t}$</td>
<td>0.0</td>
<td>17.1</td>
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Table 1 $D_E$, $D_{CL}$ and $D_{PW}$ determined from uniaxial tension along the RD. $D_{CL}$ and $D_{PW}$ are in MPa and $D_E$ is dimensionless.

<table>
<thead>
<tr>
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<th>AZ31</th>
<th>ZE10</th>
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<tr>
<td>$D_E$</td>
<td>0.1476</td>
<td>0.1119</td>
</tr>
<tr>
<td></td>
<td>(± 0.0074)</td>
<td>(± 0.0002)</td>
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<tr>
<td>$D_{CL}$</td>
<td>39.01</td>
<td>28.18</td>
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<td></td>
<td>(± 2.20)</td>
<td>(± 0.07)</td>
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<tr>
<td>$D_{PW}$</td>
<td>52.32</td>
<td>24.81</td>
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<td></td>
<td>(± 2.95)</td>
<td>(± 0.06)</td>
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Table 2 Optimized parameters of the MMC and EMMC criteria. The latter is used only for ZE10. The exponent was fixed to $n = 0.18$ for AZ31 and to 0.14 for ZE10.

<table>
<thead>
<tr>
<th></th>
<th>MMC (AZ31)</th>
<th>EMMC (ZE10)</th>
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<tr>
<td>$c_1$</td>
<td>2.824</td>
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<td>$c_2$</td>
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<td>$c_3$</td>
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<td>$c_2$</td>
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</tr>
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<td>$c_3$</td>
<td>0.889</td>
<td>2.215</td>
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Figure 1 Measured engineering stress-strain curves in (a) uniaxial tension and (b) uniaxial compression tests, (c) normalized force-notch opening displacement (NOD) in notched-bar tension tests and (d) normalized force-pin distance in shear tests for AZ31 sheets.
Figure 2 Measured engineering stress-strain curves in (a) uniaxial tension and (b) uniaxial compression tests, (c) normalized force-notch opening displacement (NOD) in notched-bar tension tests and (d) normalized force-pin distance in shear tests for ZE10 sheets.
Figure 3 Macroscopic view on the gage section of ZE10 tensile samples showing diffuse necking followed by a slanted (shear) crack; RD orientation (top) and TD orientation (bottom); scale in millimeters.

Figure 4 Details of fracture surfaces. Tensile test of AZ31, TD orientation (a) and notched bar test of ZE10, TD orientation (b) showing facets surrounded by shallow dimples originating from micro-plasticity.
Figure 5 FE meshes for (a) the notched bar and (b) shear specimens.

Figure 6 Distributions of triaxiality and Lode angle parameter at fracture along the centerline of NB specimen for AZ31: (a) RD and (b) TD. The Lode angle parameter for plane stress is calculated using Eq. 19. Similar plots for accumulated effective strains, $\varepsilon_g$ (solid) and $\varepsilon_t$ (dotted): (c) RD and (d) TD.
Figure 7 Distributions of triaxiality and Lode angle parameter at fracture along the centerline of NB specimen for ZE10: (a) RD and (b) TD. The Lode angle parameter for plane stress is calculated using Eq. 19. Similar plots for accumulated effective strains, $\bar{\varepsilon}_g$ (solid) and $\bar{\varepsilon}_t$ (dotted): (c) RD and (d) TD.
Figure 8 Distributions of triaxiality and Lode angle parameter at fracture along the centerline of SH specimen for AZ31: (a) RD and (b) TD. The Lode angle parameter for plane stress is calculated using Eq. 19. Similar plots for accumulated effective strains, $\bar{\varepsilon}$ (solid) and $\tilde{\varepsilon}$ (dotted): (c) RD and (d) TD. $X$ refers to the coordinate in the deformed configuration.
Figure 9 Distributions of triaxiality (solid) and Lode angle parameter at fracture along the centerline of SH specimen for ZE10: (a) RD and (b) TD. The Lode angle parameter for plane stress is calculated using Eq. 19. Similar plots for accumulated effective strains, $\varepsilon_{g}$ (solid) and $\varepsilon_{t}$ (dotted): (c) RD and (d) TD. X refers to the coordinate in the deformed configuration.

Figure 10 Evolutions of (a) triaxiality and (b) Lode angle parameter as a function of total effective strain ($\varepsilon_{g} + \varepsilon_{t}$) for critical elements in UT, NB and SH simulations for AZ31. Black symbols indicate failure points.
Figure 11 Evolutions of (a) triaxiality and (b) Lode angle parameter as a function of total effective strain ($\bar{\varepsilon}_g + \bar{\varepsilon}_t$) for critical elements in UT, NB and SH simulations for ZE10. Black symbols indicate failure points.

Figure 12 Dependency of the local failure strain on triaxiality and Lode angle parameter. The vertical planes represent states of constant triaxiality (a) and Lode angle parameter (b).
Figure 13 Evaluation of the (a) effective strain, (b) Cockcroft-Latham, (c) plastic work and (d) MMC criteria under different stress states for AZ31. The solid circles indicate that the corresponding data were used to determine the parameter(s) of the criterion.
Figure 14 Evaluation of (a) effective strain, (b) Cockcroft-Latham, (c) plastic work, (d) MMC and (e) EMMC criteria under different stress states for ZE10. The solid circles indicate that the corresponding data were used to determine the parameter(s) of the criterion.
Figure 15 FE models for (a) compression and (b) three-point-bending of square tube. Only a half geometry was considered in (b) considering the symmetry condition.

Figure 16 Measured and predicted force-displacement responses in square tube compression along (a) RD and (b) TD for AZ31. The predicted fracture points are indicated as symbols. VM refers to a simulation using the von Mises yield criterion.

Figure 17 Measured and predicted force-displacement responses in square tube compression along (a) RD and (b) TD for ZE10. The predicted fracture points are indicated as symbols. VM refers to a simulation using the von Mises yield criterion.
Figure 18 Distribution of accumulated damage indicator following the MMC criterion at predicted failure limit in (a) tube compression and (b) three-point-bending for AZ31 square tubes. In the latter, the cylindrical punch is not shown in the figure for clarity. The longitudinal direction of the tube is parallel to the RD in both figures.

Figure 19 Measured and predicted force-displacement responses in three-point-bending of square tubes along (a) RD and (b) TD for AZ31. The predicted fracture points are indicated as symbols.
Figure 20 Measured and predicted force-displacement responses in three-point-bending of square tubes along (a) RD and (b) TD for AZ31. The predicted fracture points are indicated as symbols.

Figure 21 Cracked samples after three-point bending testing. Left: AZ31, showing two cracks at each side; Right: ZE10, with only one crack propagating at each side in axial direction forming bulged shoulders of sheet metal on the vertical sections.
**Figure 22** Stress paths of selected finite elements in tube compression (COMP) and three-point-bending (3PB) of ZE10 square tube. The locations of the TOP and SIDE elements are indicated in Fig. 18. The fracture criteria were calibrated without information in the shaded region.

**Figure 23** Damage accumulation in uniaxial tension and compression using (a) effective strain, (b) Cockcroft-Latham, (c) plastic work and (d) MMC criteria for AZ31. The solid circles indicate that the corresponding data were used to determine the parameter(s) of the criterion.
Figure 24 Predicted cross-sectional profiles of square tubes at punch displacement of 12.04 mm in three-point-bending.

Figure 25 Evolution of (a) triaxiality and Lode angle parameter and (b) $D_{MMC}$ and effective glide strain as function of punch displacement in three-point-bending.
Figure A2 Measured and predicted force-displacement responses in square tube compression along the RD for ZE10. The predicted fracture points are based on the total strain rather than the glide strain. The Figure compares to Fig. 17.