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Lode Parameter Dependence and Quasi-Unilateral Effects in Continuum Damage Mechanics: Models and Applications in Metal Forming

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Abstract. This work concerns with two successive modifications of the Lemaitre's damage model to meet the requirements of formability prediction for today's modern steels. The first one is the quasi-unilateral damage evolution which modifies the damage driving force by scaling the elastic energy release rate due to compressive principal stress components. The second one is the shear modification by which the damage rate is multiplied by a normalized maximum shear stress dependent factor. With the assumption of non-rotating principal axes of deformation, proportional strain paths and rigid plasticity, closed form expressions for the isochronous fracture surfaces are derived for each model variant and resulting surface plots at various spaces are compared. The findings show that the former modification not only remedies the pathological reflective symmetry of the fracture surface across the Π plane with vanishing stress triaxiality ratio, but also allows hindering fracture under uniaxial compression. The latter modification by adding a direct Lode parameter dependence to the damage evolution function allows prediction of premature fracture at generalized shear stress states, a condition observed for certain advanced high strength metallic sheets. Parameter calibration is realized for each model variant using the experimental data from the literature. It is shown that the fracture model with both the shear modification and the quasi-unilateral enhancement shows the best fitting quality. Finally, the models are implemented as user subroutines for ABAQUS/EXPLICIT and used in prediction of initiation and propagation of cracks for a series of deep-drawing punch tests. A good agreement with the outputs reported in the literature is observed in terms of the shear damage occurrence zones as well as corresponding punch force-displacement diagrams.

Introduction

Low triaxiality ductile fracture engaged considerable interest in the theoretical and the experimental solid mechanics communities since the pioneering study of Bao and Wierzbicki which shows the non-monotonic dependence of the equivalent fracture strain on triaxiality [2]. Theoretical developments involved, on one side, micro-void dynamics and interaction based descriptions of the observed phenomenon, on the other side, proposed new constitutive models; see, e.g., [6], or the modification of the existing ones which are known to be working well for moderate to high triaxiality ratios (i.e. void growth dominant ductile fracture); see, e.g., [7]. In these proposals the Lode parameter (or an equivalent measure which distinguishes the axi-symmetric or shear nature of the loading) is frequently devised as an additional degree of freedom in addition to the triaxiality and equivalent plastic strain rate. The experimental studies mainly concentrate on proposing special specimen geometries for set ups aiming at incipient fracture under shear stress states.

With this point of departure, the current study aims at introducing two successive modifications to the Lemaitre damage model [4] in order to supply sufficient functional flexibility that will allow accurate predictions for a wide range of stress triaxialities. More specifically, the initial quasi-unilateral improvement prevents the fracture strains under the compressive stresses from being underestimated whereas the latter accounting for Lode dependence aims at improving the fracture strain estimates under generalized shear stress states. Besides critical assessment of the limitations and the predictive capabilities of the base model and the models with introduced enhancements in the context of metal forming practice the present paper presents analyses of the geometry of the analytically developed isochronous fracture surfaces as well as finite element simulations aiming at fracture prediction in sheet metal forming.

Consistently assuming **a**, **b**, and **c** as three second-order tensors, together with the Einstein's summation convention on repeated indices, $\mathbf{c} = \mathbf{a} \cdot \mathbf{b}$ represents the single contraction product with $c_{ik} = a_{ij} b_{jk}$. $d = \mathbf{a} : \mathbf{b} = a_{ij} b_{ij}$ represents the double contraction product, where d is a scalar. $\text{dev}\,(\mathbf{a}) = \mathbf{a} - [1/3]\text{tr}(\mathbf{a})\,\mathbf{1}$ and $\text{tr}\,(\mathbf{a}) = a_{ii}$ stand for the deviatoric part of and trace of **a**, respectively, **1** denoting the identity tensor. $\text{sym}\,(\mathbf{a})$ and $\text{skw}\,(\mathbf{a})$ denote symmetric and skew-symmetric portions of **a**. $\dot{\mathbf{a}}$ gives the material time derivative of **a**. $\langle x \rangle = [1/2]\,[x + |x|]$ describes the ramp function. The norm of **a** is denoted by $|\mathbf{a}| = \sqrt{\mathbf{a} : \mathbf{a}}$.

Stress States

A point on von Mises yield locus can be represented in terms of different parametrizations the straightforward one being with the Cartesian coordinates $(\sigma_1, \sigma_2, \sigma_3)$ of the Haigh-Westergaard stress space. A particularly useful parametrization is with the cylindrical polar coordinates (r, θ, η). As depicted in Figure 1, $r = \sqrt{2/3}\sigma_{\text{vMises}}$ represents the radius of the circle and the Lode angle $0 \leq \theta \leq \pi/3$ is measured from the axis representing axisymmetric tension. For this range the stress principals are ordered $\sigma_1 \geq \sigma_2 \geq \sigma_3$. $\eta := p/\sigma_{\text{vMises}}$ is the stress triaxiality ratio as the coordinate normal to the deviatoric plane. Normalization of the Lode angle gives $\overline{\theta}$ with the range $-1 \leq \overline{\theta} \leq 1$

$$\overline{\theta} = 1 - \frac{6\theta}{\pi}. \tag{1}$$

With reference to Figure 1 and using the even function property $\cos(\theta) = \cos(-\theta)$ principal deviatoric Cauchy stress components s_A read

$$s_\nu = \frac{2}{3}\sigma_{\text{vMises}} \cos\left(\frac{2\,[\nu - 1]}{3}\pi - \theta\right) \quad \text{for } \nu = 1, 2, 3 \tag{2}$$

with $\cos(-\theta) + \cos([2/3]\pi - \theta) + \cos([4/3]\pi - \theta)$ vanishing identically. The principal Cauchy stress components using $\sigma_\nu = s_\nu + p$ and the definition of the stress triaxiality ratio $\eta := p/\sigma_{\text{vMises}}$ read

$$\sigma_\nu = \sigma_{\text{vMises}} \left[\eta + \frac{2}{3}\cos\left(\frac{2\,[\nu - 1]}{3}\pi - \theta\right)\right] \quad \text{for } \nu = 1, 2, 3. \tag{3}$$

Finally the Lode parameter $-1 \leq L \leq 1$ is defined as

$$L = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3}. \tag{4}$$

With the ordered set of principal stresses $\sigma_1 \geq \sigma_2 \geq \sigma_3$, the definition of a shear state of stress can also be most naturally done through normalized maximum shear stress, $2[\tau_{\text{max}}/\sigma_{\text{vMises}}]$ where $\tau_{\text{max}} = [1/2][\sigma_1 - \sigma_3]$.

Lemaitre Based Fracture Criteria

We start with presenting an uncoupled version of Lemaitre's damage model by ignoring the state coupling of damage and elasticity, also the kinematic coupling damage and plasticity. We define the

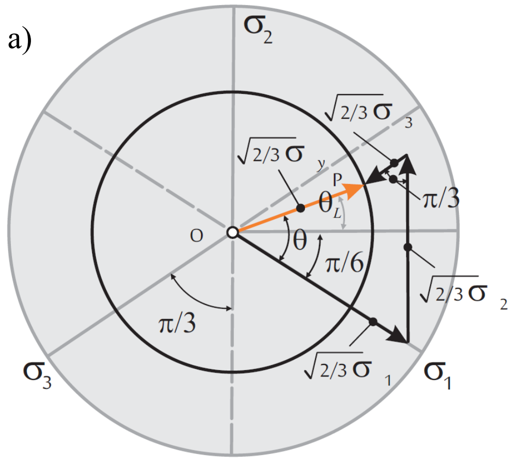

Fig. 1: Geometrical representation of the Lode angle θ, $\sigma_y = \sigma_{\text{vMises}}$ and a) projections of the von Mises yield locus as well as of point P on the (deviatoric-stress) $\Pi-$plane and with corresponding projected stress components $(\sigma_1, \sigma_2, \sigma_3)$ and b) the principal deviatoric strain components (e_1^p, e_2^p, e_3^p) of point P on the deviatoric-strain plane, note that $e_1^p + e_2^p + e_3^p = 0$.

von Mises stress $\sigma_{\text{vMises}} = \sqrt{[3/2]\,[\mathbf{s} : \mathbf{s}]}$ with $\mathbf{s} = \text{dev}(\boldsymbol{\sigma})$ with $\boldsymbol{\sigma}$ denoting the Cauchy stress tensor. Denoting the yield stress as $\sigma_y := \sigma_0 + q$ with the non-rotating axes of deformation one has the following flow rule in terms of the rate of logarithmic plastic strain $\dot{\mathbf{e}}^p$

$$\dot{\mathbf{e}}^p = \dot{\alpha^p}\,\frac{3}{2}\frac{\mathbf{s}}{\sigma_{\text{vMises}}}, \tag{5}$$

with $\text{tr}(\dot{\mathbf{e}}^p) = 0$. Letting α^p represent the equivalent plastic strain the evolution of the Lemaitre type damage indicator D is given as

$$\dot{D} = \dot{\alpha^p}\left\langle \frac{Y - Y_0}{S} \right\rangle^m \frac{1}{[1-D]^\beta}. \tag{6}$$

where Y is the damage driving force, i.e. the elastic energy release rate, and Y_0 is its threshold. For isotropic plasticity with proportional strain paths the ratio $s_\nu/\sigma_{\text{vMises}}$ is constant and the principal logarithmic plastic strains read

$$e_\nu^p = \alpha^p \frac{3}{2}\frac{s_\nu}{\sigma_{\text{vMises}}} \text{ for } \nu = 1,2,3, \tag{7}$$

where $\sigma_{\text{vMises}} = \sigma_y = C[\alpha^p + \alpha_0^p]^n$ with

$$\alpha^p = \sqrt{[2/3]\left[[e_1^p]^2 + [e_2^p]^2 + [e_3^p]^2\right]}. \tag{8}$$

With the assumption for small elastic strains as compared to the plastic ones we have $\mathbf{e}^p \approx \mathbf{e}$. Representing the damage driving force in terms of principal Cauchy stresses σ_ν for $\nu = 1,2,3$ we have

$$Y = \frac{1}{4\mu}\left[C[\alpha^p + \alpha_0^p]^n\right]^2 f(\eta, \theta), \tag{9}$$

with the plastic flow condition $C[\alpha^p + \alpha_0^p]^n = \sigma_{\text{vMises}}$ and

$$f\left(\eta,\theta\right):=\sum_{\nu=1}^{3}\left[\eta+\frac{2}{3}\cos\left(\frac{2\left[\nu-1\right]}{3}\pi-\theta\right)\right]^{2}+\left[\frac{2\mu}{\kappa}-3\right]\eta^{2}. \tag{10}$$

Then the fracture strain α_f^p is computed as

$$\alpha_f^p=\alpha_f^p(\eta,\theta)=B\left[\frac{2}{3}+\frac{2\mu}{\kappa}\eta^2\right]^{\frac{-m}{2mn+1}}-\alpha_0^p \tag{11}$$

where

$$B=\left[\frac{[2mn+1]}{[\beta+1]}\left[\frac{4\mu S}{C^2}\right]^m\right]^{\frac{1}{2mn+1}}. \tag{12}$$

As seen the model does not account for an explicit Lode parameter dependence.

Modification I: Quasi-Unilateral Damage Evolution

A modification based on the quasi-unilateral conditions could be introduced modifying the damage driving force Y, [3], which eventually gives the strain at fracture as

$$\alpha_f^p=B\left[f^{\oplus}(\eta,\theta)\right]^{\frac{-m}{2mn+1}}-\alpha_0^p \tag{13}$$

with

$$f^{\oplus}\left(\eta,\theta\right)=f^{+}\left(\eta,\theta\right)+hf^{-}\left(\eta,\theta\right), \tag{14}$$

Here the quasi-unilateral damage evolution parameter $h\in[0,1]$ acts as a weighting factor which scales the energetic contribution of the compressive principal stresses and the hydrostatic stress. Also $f^{+}\left(\eta,\theta\right)$ and $=f^{-}\left(\eta,\theta\right)$ are defined as

$$\left.\begin{array}{lll} f^{+}\left(\eta,\theta\right) & = & \sum_{\nu=1}^{3}\left\langle\eta+\frac{2}{3}\cos\left(\frac{2(\nu-1)}{3}\pi-\theta\right)\right\rangle^{2}+\left[\frac{2\mu}{\kappa}-3\right]\langle\eta\rangle^{2} \\ f^{-}\left(\eta,\theta\right) & = & \sum_{\nu=1}^{3}\left\langle-\eta-\frac{2}{3}\cos\left(\frac{2(\nu-1)}{3}\pi-\theta\right)\right\rangle^{2}+\left[\frac{2\mu}{\kappa}-3\right]\langle-\eta\rangle^{2}. \end{array}\right\} \tag{15}$$

Modification II: Shear Modification

This enhancement follows in the lines of empirical multiplicative modification of the Oyane's fracture criterion, [8] by [6] where the damage evolution is modified to account for the normalized maximum shear stress which renders

$$h(\theta)=\frac{2\tau_{\text{max}}}{\sigma_{\text{vMises}}}=\frac{2}{3}\left[\cos\left(\theta\right)-\cos\left(\frac{4}{3}\pi-\theta\right)\right]. \tag{16}$$

This leads to the following immediate integration for the equivalent fracture strain with the assumption of radial loading paths

$$\alpha_f^p=B\left[h(\theta)^{\delta}f^{\oplus}(\eta,\theta)^{m}\right]^{\frac{-1}{2mn+1}}-\alpha_0^p. \tag{17}$$

Applications

In this part we give first the calibration of the model parameters for TRIP690 and then use these parameters in finite element analysis of shear fracture development during rectangular deep drawing. The simulations are realized in ABAQUS/EXPLICIT together with implemented as user defined material subroutines.

The material studied falls under the group of Advanced High Strength Steels. As a product of ThyssenKrupp Steel Europe AG, the sheets are cold rolled Retained Austenite Steel where the minimum tensile strenght is 690MPa, hence TRIP690. Since the tests are realized on the sheets the tests are reported as to correspond to the plane stress condition. Anisotropy of the material in both plasticity and fracture is reported as negligible which makes it possible to use the current modeling framework. For TRIP690 the shear modulus is $\mu = 80769.2$ MPa and the bulk modulus is $\kappa = 175000$ MPa. For the plastic hardening of the material a power hardening rule is used with $C = 1275.9$ MPa, $\alpha_0 = 0$ and $n = 0.2655$. For fracture calibration, tests are conducted on five types of specimens: dog-bone specimen, flat specimen with cutouts, punch test, butterfly specimen in tension and butterfly specimen in simple shear. The fracture strains are computed using finite element analysis with shell elements and like in the previous example represented in terms of the average triaxiality and Lode parameter $(\eta_{\text{ave}}, \overline{\theta}_{\text{ave}})$ values listed in [5] and [1]. The summary of the material parameters identified are listed in Table 1.

Table 1: Determined parameters of the selected fracture criteria for TRIP690.

Model	Parameters
Lemaitre mod II.	$\beta = 1$; $m = 0.2132$; $S = 30.93$; $h = 0$; $\delta = 4.579$
Lemaitre mod I.	$\beta = 1$; $m = 2.448$; $S = 2.247$; $h = 1$
Lemaitre	$\beta = 1$; $m = 2.448$; $S = 2.247$

A 2D plot of the fracture criteria in η space is possible assuming plane stress conditions. For the case of plane stress stress states, one has a direct relation between $\overline{\theta}$ and the triaxiality η as

$$\overline{\theta} = 1 - \frac{2}{\pi} \arccos\left(-\frac{27}{2}\eta\left(\eta^2 - \frac{1}{3}\right)\right). \tag{18}$$

In Figure 2 the equivalent fracture strain is given in the triaxiality space within the range of $-\frac{1}{3} \leq \eta \leq \frac{2}{3}$ for the reported variants of Lemaitre fracture criteria. The experimental points are also shown as red dots. As seen, Lemaitre model modification II has considerable improvement in terms of flexibility to represent nonmonotonic dependence of the fracture strain on triaxiality.

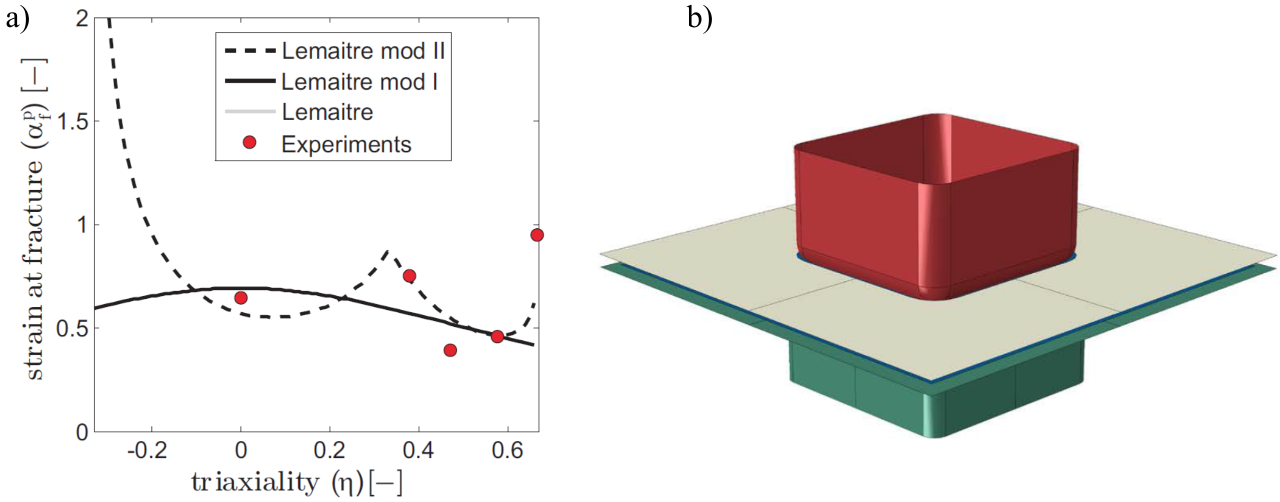

Fig. 2: a) Plot of the fracture locus for TRIP690 for the best four fracture criteria represented at the triaxiality (η) space for plane stress conditions for the identified parameters. b) The finite element model setup of the rectangular deep drawing test.

In Figure 3 it is shown that maximum damage indicator takes place in the drawing region where cracking occurs under in-plane maximum shear stresses using the model with both enhancements. Corresponding force-displacement diagram given in the same figure shows that there is a good agreement between the punch force displacement diagrams handled in the simulations and experiments reported in [5].

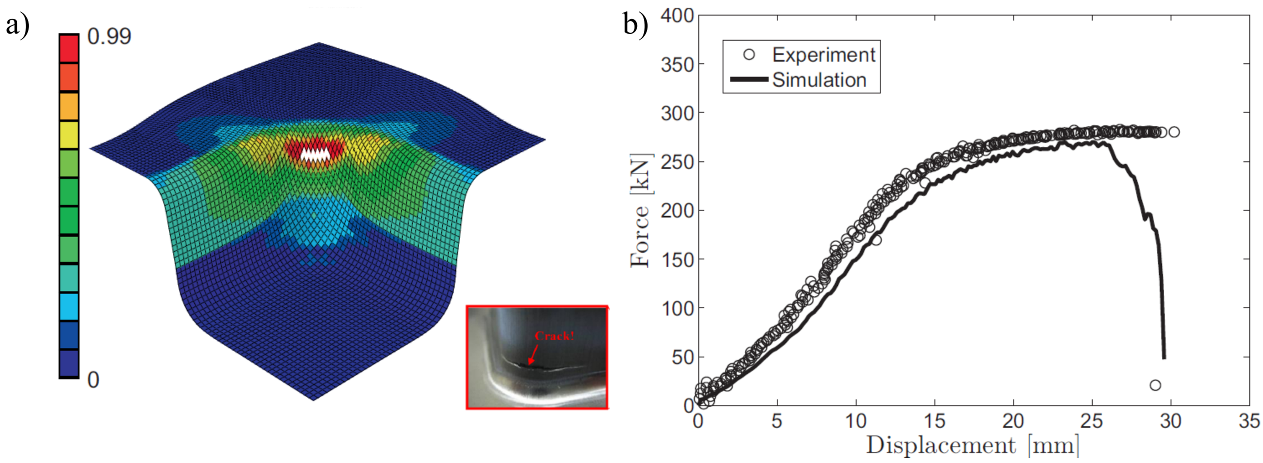

Fig. 3: a) Damage distribution and crack formation in the drawing region due to shear damage accumulation. The small figure shows experimentally determined shear cracks in the same region during rectangular deep drawing of TRIP690 [5]. b) The comparison of the simulation result with the experimentally determined [5] punch force-displacement diagram.

Conclusions

Building upon the classical Lemaitre's damage model we present two rather empirical enhancements with the least number of material parameters. The former is the already known quasi-unilateral damage evolution. This enhancement relies on a weighted damage evolution for the compressive principal stress components which is overestimated by the conventional model. The latter is a novel suggestion to remedy shear fracture. The blend of quasi-unilateral and shear modification seems to give promising results as the experimental calibration studies as well as simulation result comparisons with the experimentally determined ones show.

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