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Remarks on the Green–Naghdi theory of heat conduction

Swantje Bargmann

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Abstract

The Green–Naghdi theory of heat conduction enjoys great research interest because it is developed in a very general way and because it is capable of accounting for thermal pulse transmission in a very general manner. In this paper, that theory is revisited, and some questions it leaves open are pointed out.

1 Introduction

Fourier's constitutive equation for the heat flux is well known; usually, heat conduction is based on this law. Most engineering applications are described accurately by the ensuing parabolic heat equation, but material behavior at cryogenic temperatures can completely differ from that at room temperature. One of the properties that might change is the way heat propagates. Predictions of the classical theory may diverge conspicuously from experimental data. Moreover, Fourier's law implies the unphysical property that temperature perturbations propagate at infinite speed. In order to overcome this drawback, intensive research activities have led to modified theories of heat conduction which in general are hyperbolic. The detection of second sound[1] has intensified the development of non-classical heat conduction theories.

[1] The phenomenon of second sound refers to the fact that wave-like energy transport takes place, i.e., heat propagates under the form of thermal waves. Until now, second sound in solids has been observed in very pure crystals in the vicinity of the local →

A generalized theory was introduced by Green and Naghdi [28] and a number of studies devoted to its investigation have already been released. Although the papers on non-classical thermoelasticity by Green and Naghdi were published shortly before their deaths, today they are the most cited publications – more than 300 citations (according to Web of Science) of their theory of thermoelasticity without energy dissipation [30], with a steadily increasing number of citations each year.

The Green–Naghdi approach is capable of accounting for thermal pulse transmission and is embedded in rational thermodynamics. The theory is subdivided into three types, labeled Types I, II, and III. A wide range of heat flow problems can be modeled. The novelty of their approach is the introduction of the so-called thermal displacement, a field whose time derivative is the empirical temperature.

The non-classical Type II (Fourier's classical theory (Type I) is contained as well) exhibits the outstanding property of being non-dissipative. In other words, heat conduction no longer necessarily implies internal dissipation. The resulting heat equation is hyperbolic. In particular, thermal wave propagation at finite speed is incorporated. Furthermore, the entropy flux vector is determined in terms of the same potential as the mechanical stress and is proportional to the thermal displacement gradient.

Type III is a general extension and is capable of modeling according the classical Fourier theory as well as undamped thermal wave propagation – and, in addition, many more phenomena.

Further physical, mathematical, and numerical studies are still needed to clarify the applicability, the secrets, the capabilities, as well as the limits of the Green–Naghdi theory. This contribution is in this spirit and aims at highlighting the distinct features of the Green–Naghdi approach and addressing open questions.

2 Green and Naghdi's theory of thermoelasticity

The basic feature of Green and Naghdi [28] is the definition of the thermal displacement α. It is the time integral over the empirical temperature T:

$$\alpha(\boldsymbol{X}, t) := \int_{t_0}^{t} T(\boldsymbol{X}, \mathrm{t})\, \mathrm{dt} + \alpha_0. \tag{1}$$

thermal conductivity maximum at cryogenic temperatures [5, 34, 40, 59, 66]. Furthermore, a wavy nature of heat propagation has been detected in superfluids and in certain inhomogeneous materials [62].

Here, $\boldsymbol{X} \in \mathbb{R}^3$ denotes the position and $t \in \mathbb{R}$ the time; α_0 is the initial value of the thermal displacement α at the reference time t_0; $\dot{\alpha} = T$ holds.

It is well known that the classical form of the heat equation for a stationary rigid heat conductor reads

$$\rho c \dot{T} = -\operatorname{div} \boldsymbol{q} + r. \tag{2}$$

Here, the constant c is the specific heat at constant pressure, t is the time, and $\boldsymbol{q}$ denotes the heat flux vector. r is the heat supplied to the body by the external world. Furthermore, the standard assumption $\boldsymbol{q} = \theta \boldsymbol{h}$ relating the heat flux vector $\boldsymbol{q}$ to the entropy flux vector $\boldsymbol{h}$ holds. In the classical theory, Eq. (2) is retrieved from the energy balance.

Instead of using the energy balance as a basis, Green and Naghdi base their theory on the entropy balance. At a later point, they substitute the energy balance into the entropy balance. Thus, the role of entropy and energy is in a sense switched with respect to the classical approach. The Green–Naghdi theory is subdivided into three different types of heat conduction, which rely on the introduction of three different state spaces. In the case of Type I, the constitutive quantities are assumed to depend on the temperature T and on its gradient ∇T. The free energy ψ is assumed to be standard,

$$\psi_1 = \psi_{\text{standard}}, \tag{3}$$

where examples for a standard thermal free energy are, e.g., $\psi_{\text{standard1}} = c[T - T \ln T]$ or $\psi_{\text{standard2}} = -c[T - T_0]^2/[2T_0]$. The second one results from the Taylor expansion of the first and is used by Green and Naghdi [28] for linearization in order to obtain a Fourier type heat equation. The heat flux follows Fourier's constitutive equation, i.e.,

$$\boldsymbol{q}_1 = -\kappa_1 \nabla T. \tag{4}$$

Here, κ_1 represents the thermal conductivity and Eq. (4) leads to a parabolic temperature equation,

$$\rho c \dot{T} = \operatorname{div}(\kappa_1 \nabla T) + r. \tag{5}$$

Green and Naghdi's approach differs significantly from the classical way in the derivation of the heat equations. Usually, the heat equation is derived via the energy balance. In turn, Green and Naghdi use the entropy balance as a basis, and elevate it to the most important balance equation in their theory. As a result, the heat equations of the classical and of the Type I theory look the same from the mathematical point of view. However, the

physical meaning is not the same. In the classical approach, the heat equation is a consequence of the energy equation, whereas in the Green–Naghdi approach it is a consequence of the entropy equation.

A point in the state space of Type II is a triplet consisting of the temperature T, the thermal displacement α, and the thermal displacement gradient $\nabla\alpha$. The free energy ψ is the sum of a standard and a non-standard contribution:

$$\psi_2 = \psi_{\text{standard1}} + \frac{1}{2}\kappa_2 \nabla\alpha \cdot \nabla\alpha. \tag{6}$$

The constitutive relation for the entropy flux $\boldsymbol{h}$ is given by

$$\boldsymbol{h}_2 = -\frac{\partial\psi}{\partial\nabla\alpha} = -\kappa_2\nabla\alpha. \tag{7}$$

The resulting hyperbolic heat equation,

$$\rho c\dot{T} = \rho c\ddot{\alpha} = \operatorname{div}(\kappa_2\nabla\alpha) + r, \tag{8}$$

represents a wave equation for $r = 0$ and constant $\kappa_2 > 0$. Temperature is propagated without damping at a wave speed of $\sqrt{\kappa_2/[\rho c]}$.

The Type III theory is the most general one. The state space of Type III consists of α, T, $\nabla\alpha$, and ∇T. In the case of Type III, the free energy function is assumed to be

$$\psi_3 = \psi_{\text{standard1}} + \frac{\kappa_3}{2}\nabla\alpha \cdot \nabla\alpha \tag{9}$$

(see Ref. [29]), and the heat flux is expressed as follows:

$$\boldsymbol{q}_3 = -\kappa_3\nabla\alpha - \kappa_4\nabla T. \tag{10}$$

The heat equation reads

$$\rho c\dot{T} = \rho c\ddot{\alpha} = \operatorname{div}(\kappa_3\nabla\alpha) + \operatorname{div}(\kappa_4\nabla T) + r. \tag{11}$$

3 Distinct features of the Green–Naghdi theory

Green and Naghdi derived their theory in a very general and thermodynamically consistent way. After formulating the balance laws, applying material modeling principles, exploiting the second law of thermodynamics, and formulating constitutive assumptions, the governing equations were derived.

Probably the most distinct and outstanding feature is the theory of Type II, which in general (see also Section 4.7) excludes internal dissipation. As a consequence, it is possible to arrive at a physically meaningful variational formulation [58], whereas classical thermoelasticity is not deducible from a variational principle. The governing equations of Type II correspond to the Euler–Lagrange equations of thermoelasticity and the latter lead to the solutions of the unknown mechanical and thermal fields. In this context, the existence of a potential for different modeling quantities is the key characteristic.

This construction of Type II theory leads to the outstanding property that the entropy flux $\boldsymbol{h}$ (and, thus, the heat flux $\boldsymbol{q}$) is determined by the free energy. Consequently, the entropy η, the entropy flux $\boldsymbol{h}$ and the mechanical stress tensor (in the case of a coupled theory) are determined by the same potential. This is elegant from a material modeling point of view because all relevant modeling quantities are determined by one and the same scalar functional. In the classical theory, constitutive equations have to be stated for both the free energy and the heat flux. The constitutive relation for the Type II heat flux naturally follows from the Green–Naghdi theory.

Furthermore, the Green–Naghdi theory overcomes the drawback of Fourier's theory of infinite propagation speed. In Green–Naghdi thermoelasticity, heat pulses are permitted to travel as thermal waves of finite speed.

4 Remarks

4.1 Entropy balance

When it comes to entropy, it is possible to define an entropy inequality or an entropy balance including a non-negative internal entropy production. The latter path was chosen by Green and Naghdi [28]. It is often misinterpreted by several authors (due to clumsy wording by Green and Naghdi), who state that Green and Naghdi do not make use of the second law of thermodynamics. However, they do assume a non-negative entropy production, i.e., they do apply the second law.

Green and Naghdi postulated the entropy balance for the first time in their work from 1977 [27], i.e., much earlier than their theory of thermoelasticity; as noted by them in [30], with only limited motivation back then. In Green and Naghdi's theory of thermoelasticity, the entropy balance in combination with the second law of thermodynamics is used to obtain restrictions on the constitutive quantities. Precisely, they require that

the reduced energy equation holds as an identity for all thermomechanical processes. As noted in [30], their procedure differs significantly from the common Clausius–Duhem approach, which uses an entropy imbalance.

4.2 Entropy – heat flux relation $\boldsymbol{q = \theta h}$

In all three cases, Green and Naghdi apply the assumption that the entropy flux $\boldsymbol{h}$ is equal to $\boldsymbol{q}/\theta$. θ is an absolute temperature for some materials, but not necessarily for all; see Section 4.4. The relationship $\boldsymbol{q} = \theta\boldsymbol{h}$ is not universally true.[2] Due to the introduction of the thermal displacement α the relationship cannot be transferred directly to the non-classical theory. Green and Naghdi are aware of this fact [28, Section 10] but do not provide the proof that the assumption $\boldsymbol{q} = \theta\boldsymbol{h}$ holds for all three cases of their theory. The contribution [12] closes this gap for the important class of isotropic materials. Furthermore, it is shown in [12] that in most cases θ can be identified with the absolute temperature.

4.3 The thermal displacement α

The thermal displacement α is a scalar macroscopic quantity. As stated by Green and Naghdi [28], it can be regarded as representing, on the molecular scale, some "mean" thermal displacement magnitude. α and $\nabla\alpha$ are taken as independent constitutive variables.

Green and Naghdi were not the first to come up with the idea of a variable like the thermal displacement. Helmholtz [35] introduces a Lagrangian coordinate that exists only through its time derivative. In 1921, von Laue [50] uses the concept of thermacy (later on, van Dantzig [22] called it thermasy), which equals the thermal displacement, the only difference being the sign (thermacy $= -\alpha$).

A similar idea has also been utilized by Gurtin and Pipkin [33], but in a way different from Green and Naghdi's. Although their construction yields a heat flux $\boldsymbol{q}$ which is determined by the functional for the free energy ψ, the latter does not play the role of a potential as is the case in the theory of Green and Naghdi. Gurtin and Pipkin's [33] constitutive relation for the heat flux reads

$$\boldsymbol{q} = -\int_0^\infty a(s)\nabla\theta(t-s)\,\mathrm{d}s, \tag{12}$$

[2] The possible inappropriateness of the Clausius–Duhem approach in the case of mixtures, polar continua, structured continua, and coupled field gradient theories is mentioned in, e.g., Refs. [37,48,86], whose authors suggest exploiting the entropy principle according to Müller and Liu [54,64], as it is the most general approach.

where $a(s)$ is a differentiable scalar-valued function referred to as the heat-flux relaxation function.

As mentioned by Podio-Guidugli [70], one can think of the thermal displacement α as a fast variable in the sense that only its time derivative, i.e. the temperature T, is observable on the macroscopic time scale. Kalpakides and Dascalu [45] interpret α as a continuous representation of the lattice vibration and therefore as a phenomenon of quantum-mechanical origin. Van Dantzig [22] interprets α as a thermal time.

4.4 The absolute temperature θ in the Green–Naghdi theory

Green and Naghdi [28] define α via the empirical temperature T. Most researchers utilize the same assumption. However, some (e.g., Ref. [43]) assume that the thermal displacement is a function of the absolute temperature, as done in Refs. [22, 33]. In most cases, both assumptions lead to results which only differ by a proportionality factor.

In Ref. [28] Green and Naghdi define the temperature θ in terms of the empirical temperature T and the thermal displacement α, i.e., $\theta := \theta(T, \alpha)$. θ is an absolute temperature for some materials, but not necessarily for all. In what is Eq. (8.22) of [28] they specify a response function for θ as follows:

$$\theta = a + bT + d_1\alpha, \tag{13}$$

with $a > 0$, $b > 0$ and d_1 being constants. In a later work [30], they allow the absolute temperature to depend also on the thermal displacement gradient $\nabla\alpha$.

4.5 Analogies

The thermal displacement α received its name in order to express the role analogy with the mechanical displacement. This terminology stems from statistical mechanics, where temperature is assumed to be proportional to the mean kinetic energy of a particle system, i.e., T is associated with a (mean) velocity. Consequently, one can think of looking at entropy η as a thermal momentum (in analogy with the mechanical momentum), see, e.g., Ref. [70].

4.6 The free energy ψ

In the case of the Type II and III theories, the free energy ψ is stated to consist in additional terms being quadratic in the thermal displacement gradient

$\nabla\alpha$; see Eqs. (6) and (9). This becomes necessary because in the Type II theory the entropy flux is determined via $\boldsymbol{h} = -\frac{\partial\psi}{\partial\nabla\alpha}$. Hence, the free energy must depend on the thermal displacement gradient. However, this leads to the question of what this does to the absolute value of the free energy, because this now depends on the choice of the material parameter κ_2. The heat equation of Type II is perfectly suitable to model the second sound phenomenon, which is only observed in a small temperature range. Green–Naghdi theory implies that the material's energy for κ_2 large is governed by the additional term, which, in this temperature range, may well dominate ψ_{standard}. Thus, the material's energy is significantly higher in this temperature window. Unfortunately, to the author's knowledge, there are no experimental data available to clarify this issue.

4.7 Terminology: Type II versus theory without energy dissipation

Several authors, including Green and Naghdi themselves, refer to Type II as the theory without energy dissipation. This use of the terminology implicitly suggests that this is always the case. However, the Type II theory is not necessarily dissipation free. By construction, the free energy ψ depends on the primary variables α, T, and $\nabla\alpha$ (see Ref. [28]). This results in a constitutive dependence of the internal dissipation ξ on the thermal displacement α: $\xi = -[\frac{\partial\psi_2}{\partial\alpha} + \frac{\partial\theta}{\partial\alpha}]$. Green and Naghdi chose the free energy ψ_2 to be independent of α and $d_1 = 0$. With this choice, internal dissipation ξ equals zero. However, one could also choose a free energy function depending on α or take $d_1 \neq 0$, arriving at a theory of Type II involving internal dissipation!

4.8 The conductivity coefficient κ_2

The thermal conductivity κ_1 is a material parameter measured in W/[mK] units, which is determined experimentally presuming Fourier's law of heat conduction. Green and Naghdi do not comment on how to choose a value for κ_2, κ_3, and κ_4. κ_4 has the same dimensions as κ_1 and plays the role of a thermal conductivity as well. The units of κ_2 and κ_3 are different, namely, W/[smK]. They are referred to as non-classical thermal conductivities, conductivity coefficients, material characteristics, or simply material parameters. The values of $\kappa_{2,3,4}$ depend on the problem at hand, of course. When modeling the phenomenon of second sound, the values for $\kappa_{2,3}$ can be determined in the same way the thermal conductivity κ_1 is computed. The velocity of the wave is measured in the experiments and given

by $\sqrt{\kappa_2/\rho c}$, with ρ and c known. Consequently, κ_2 can easily be calculated. Moreover, when applying Green–Naghdi theory to the second sound phenomenon, it is reasonable to assume $\kappa_2 = \kappa_3$. A conclusive experimental validation of theories of Type II and III is wanted. The thermal displacement α, its gradient $\nabla\alpha$, and the conductivity coefficient κ_2 are quantities difficult to measure with the current experimental facilities. However, their determination is important for the credibility of the theory.

4.9 Type II theory versus Type III theory

At first sight, the heat equation of type II (Eq. (8)) looks like a special case of the heat equation of type III (Eq. (11)). However, the physical meaning of the two types differs greatly. If $\kappa_4 = 0$ is zero, the heat fluxes read, respectively $\boldsymbol{q}_2 = \theta \boldsymbol{h}_2 = -\kappa_2 \theta \nabla\alpha$ and $\boldsymbol{q}_3 = -\kappa_3 \nabla\alpha$. The heat flux $\boldsymbol{q}_2$ is measured in different units ([WK]) from the classical heat flux ([W]). Consequently, the Type II theory is not a limiting case of the Type III theory. Furthermore, the heat equations of Type II and III are the same for $\kappa_4 = 0$ from a mathematical point of view, but they do not represent the same physical object due to the different heat fluxes.

It remains to be seen to what class of materials one should apply Type II and/or Type III theories. When it comes to modeling thermal waves, the heat equation of Type II and Type III are both suitable. Unfortunately, there exist no experimental data on whether second sound in solids involves internal dissipation or not. In superfluids, it has been shown that second sound waves propagate without dissipation.

4.10 Green and Naghdi's theory of dissipation-less viscous fluids

Although the thermomechanical theory for solids has received high attention, this is not the case for Green and Naghdi's transfer of their ideas to fluids. Their theory of dissipation-less viscous fluids [31] creates a new type of non-Newtonian fluids and allows for dissipation-less flows with undamped second sound heat waves. This is due to the fact that in their theory fluids have more general thermal properties than in standard approaches. To date, only very few researchers (see, e.g., Refs. [43,77–79]) have picked up this idea and applied it to viscous fluids and gases. A lot remains to be done in this area of research. In many conference discussions, scientists state that they believe that the Green–Naghdi fluid theory is "a suitable candidate for many modern applications" (e.g., [79]), such as nanofluids or a single phase theory for Helium II. However, they have not yet been able to show that this

is indeed the case.[3] Nevertheless, this author believes that it is only a matter of time before this will be caught up.

5 Conclusion

Green and Naghdi have introduced a very general theory representing an alternative candidate for modeling thermoelasticity. A wide range of heat flow problems can be modeled, such as the classical problems as well as the second sound phenomenon. Moreover, Green–Naghdi Type II theory does not exhibit the problem of infinite wave propagation speed.

The search for an all-embracing non-classical theory of thermoelasticity is not yet finished. Only a detailed investigation of the most promising approaches can lead to a value judgement; the Green–Naghdi theory certainly is one of those, in the author's opinion. In addition to theoretical analyses, a conclusive experimental validation of the theories of Type II and III is wanted.

Appendix

In the following, we shortly discuss the various research directions based on the Green–Naghdi theory, which are investigated in different research communities. None of the listings claims to be complete; there are simply too many. The book [84] surveys different theories of generalized heat wave equations, including the Green–Naghdi approach. Acceleration as well as shock waves within Green–Naghdi theory are discussed, and uniqueness, growth, and spatial decay analytical results are presented.

Mathematical results can be found, both for the linearized and the nonlinear theory [74], for Type II [16, 17, 32, 42, 47, 51, 74] and for Type III theories [18, 32, 44, 47, 51–53, 55, 60, 63, 71, 80, 88, 89], for the dynamic problem as well as for the (quasi-) static reduction [74]. Mathematicians have published papers concerning, e.g., existence of solutions [20], uniqueness results [8, 16, 19, 56, 65, 74, 75], stability properties [72–74, 76], spatial decay and wave propagation [15, 41, 42, 61, 71].

Several contributions to the analysis of Green and Naghdi's ideas stem from the community of applied mechanics. Thermodynamical investigations [12, 20, 70] are carried out, the theory is formulated in a Hamiltonian

[3] This is in contrast to the Green–Naghdi theory for solids, which has been proven (and are necessary) for many applications.

framework [57] and on the material manifold [9, 24, 45, 46, 58]. In addition, the Green–Naghdi theory is extended to more complex inhomogeneous materials. For example, a micropolar Green–Naghdi theory is the subject of Refs. [21, 25, 49, 69, 85], whereas micromorphic Green–Naghdi bodies are investigated in Ref. [39]. Multiphysics problems are studied in Refs. [2, 4, 7, 9, 23, 26, 38, 82, 83].

Although the Green–Naghdi theory is also attractive from a numerical point of view, numerical investigations are significantly fewer than the numerous theoretical contributions. Computational results are found in Refs. [1, 6, 9–11, 13, 14, 36, 67]. All of the aforementioned authors apply the Finite Element Method; further numerical results are presented in Refs. [3, 68, 81, 87].

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Author information

Swantje Bargmann, Institute of Continuum Mechanics and Material Mechanics,
TU Hamburg-Harburg, Eißendorfer Str. 42, 21073 Hamburg; and
Helmholtz-Zentrum Geesthacht, Institute of Materials Research,
Max-Planck-Str. 1, 21502 Geesthacht, Germany.
E-mail: swantje.bargmann@tu-harburg.de